Aspects of Collinearity Property in Mechanics

Răzvan Bogdan Itu, Mihaela Toderaş

Abstract: ***Interdisciplinarity encourages students to make connections between different academic disciplines, fostering a deeper understanding of complex real-world problems. By integrating various subjects, students are able to develop critical thinking skills and apply their knowledge in practical ways. This approach not only enhances their learning experience but also prepares them for the challenges they may face in their future careers. In the paper, a strong connection between mathematics and mechanics has been demonstrated. It is important to note that the discussion of this topic is just scratching the surface of the many aspects that can be explored. This example highlights the principle of continuous learning and the endless possibilities for acquiring new knowledge in any field. The process of knowledge is infinite and always open to new contributions. By integrating knowledge from different disciplines, individuals can gain a holistic understanding of complex concepts and phenomena. This interdisciplinary approach fosters critical thinking skills and encourages creative problem-solving, enabling learners to tackle real-world challenges with a broader perspective. Additionally, the collaboration between disciplines promotes innovation and encourages the development of new ideas and solutions. This paper presents aspects regarding the application of the collinearity property in mechanics. The laws of motion of a rigid body, scalar functions of time are meant, which determine, in any moment of the motion, the position of the body in relation to a benchmark through the examples taken in the study were taken from point kinematics and rigid kinematics, also studying how the velocity and acceleration of the points of the solid body vary, in relation to the same reference system.***

Keywords: ***Collinearity, Kinematic Movement, Velocity.***

I. INTRODUCTION

The problem of collinearity has been considered in many studies of applied mechanics carried out by different researchers [6], [8], [10], [11], [18]. Starting from some properties of the Lemaitre regularization [9], Titov studies the problem with three bodies. He applies these properties to the degenerate case of a rectilinear problem in the space of shapes and obtains a number of collinear orbits [13] and also to the degenerate case of an isosceles problem in the space of forms [14]. Applying the collinearity problem, the author obtains a number of collinear orbits, respectively isosceles orbits, whose properties he analyses.

Special results were obtained by Balbiani et al. [1], [2]. Starting from the fact that a geometrical figure is a relation on a finite set of points whose properties can be expressed using equations between terms of the first order, they elaborate a narrowing-based unification algorithm that will solve every system of geometrical equations in the language of affine geometry of collinearity. Advanced research has been carried out by applying the collinearity procedure to the identification of large-scale linear systems using Gaussian regression [17][24]. The authors developed a strategy cast in a Bayesian regularization framework where any impulse response is seen as realization of a zero-mean Gaussian process. Kinematics studies mechanical motion, without taking in consideration forces and moments, that is, it exclusively follows its geometric aspect. Kinematics of the point studies the movement of a material point, irrespective of the causes of this movement. It allows the study of the relationships between the parameters describing the motion and their equations or transformations in various systems of coordinates or in the case of a change of a reference system [3], [5], [12]. The movement of a body in relation to a reference system is known, if the motion laws of each point of the body can be determined. By the laws of motion of a rigid body, scalar functions of time are meant, which determine, in any moment of the motion, the position of the body in relation to a benchmark [4], [7], [15]. The movement of a solid rigid body in relation to a fixed reference system is only determined when at any time, the position, velocity and acceleration of any point of the rigid body are known. The fundamental problem of the kinematics of the rigid body lies in establishing the distribution of velocity and acceleration (determination at a certain moment of the set of velocity and acceleration vectors for their various points. In mechanics, especially in kinematics, as in geometry, there are aspects in which reference is made to collinearity property, both in theory, and in applications [16], [17][22][23].

The paper presents different aspects of the collinearity property, the theoretical solution and the applied part.

II. PROBLEM FORMULATION

A. Properties of Velocity Distribution in Rigid Bodies

In the kinematics of rigid bodies, the fundamental problem lies in establishing the distribution of velocity and acceleration. The motion of a rigid body is known when there is the possibility of knowing the movement of any of its points in relation to a certain reference system x1O1y1, assumed to be immobile. Practically, it is neither necessary nor possible for the movement of the rigid body to be described, by the movement of each of its points. Since the hypothesis of rigidity exists, namely the relative distances between the points of the rigid body stay constant, it is sufficient for the exact positions of only some of the points to be known, at any time, from which the positions of the others are determined.

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It results that it is useful to consider a reference system xOy solidary with the rigid body, in relation to which the points of the body to be positioned. Similarly, one can also study how the velocity and acceleration of the points of the solid body vary, in relation to the same reference system [19]-[21]. These variation laws for the velocity and acceleration of points, depending on their position inside the solid rigid body (and not depending on time) is called velocity distribution, and acceleration distribution, respectively. These distributions are established starting from expressing the position of a current point M of the solid body in relation to the two reference systems (Fig. 1).

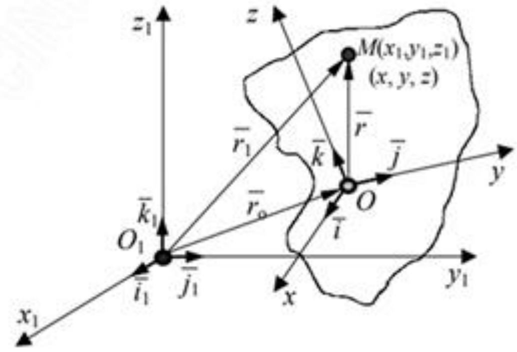

Fig. 1. The Position of the Current Point M of the Solid Body In Relation to the two Reference Systems

Thus, the position vector for point M is written:

$$\bar{r}_1(t)=\bar{r}_0(t)+\bar{r}(t)$$
$$=\bar{r}_0(t)+x\bar{i}(t)+y\bar{j}(t)+z\bar{k}(t) \quad (1)$$

The known coordinates x, y, z of the point stay constant during the motion. The unknown elements of the problem are in this case only the vectorial functions $\bar{r}_0(t)$ (the origin of the mobile system, a system solidary with the rigid body, and which obviously moves alongside with it), and the versor of the axes of the mobile system, $\bar{i}(t),\bar{j}(t),\bar{k}(t)$. Derivation in relation to time of the (1) leads to establishing the velocity of point M in relation to the fixed reference system:

$$\bar{v}_M=\dot{\bar{r}}_1(t)=\dot{\bar{r}}_0(t)+\dot{\bar{r}}(t) \quad (2)$$

The derivative of the position vector of point O in relation to time represents its velocity:

$$\dot{\bar{r}}_0(t)=\bar{v}_0(t) \quad (3)$$

And:

$$\dot{\bar{r}}(t)=x\dot{\bar{i}}(t)+y\dot{\bar{j}}(t)+z\dot{\bar{k}}(t)$$
$$\dot{\bar{r}}(t)=x\bar{\omega}\times\bar{i}+y\bar{\omega}\times\bar{j}+z\bar{\omega}\times\bar{k} \quad (4)$$
$$\dot{\bar{r}}(t)=\bar{\omega}\times(x\bar{i})+\bar{\omega}\times(y\bar{j})+\bar{\omega}\times(z\bar{k})$$
$$\dot{\bar{r}}(t)=\bar{\omega}\times\bar{r}$$

Since:

$$\dot{x}=0,\ \dot{y}=0,\ \dot{z}=0,\ \dot{\bar{i}}=\bar{\omega}\times\bar{i},\ \dot{\bar{j}}=\bar{\omega}\times\bar{j},\ \dot{\bar{k}}=\bar{\omega}\times\bar{k}$$

(Poisson equations) and is the angular velocity vector of rotation of the mobile system.

In the end (2) becomes:

$$\bar{v}_M=\bar{v}_0+\bar{\omega}\times\bar{r} \quad (5)$$

Equation (5) is known as Euler formula for velocity distribution represents in fact the fundamental formula of the kinematics of rigid bodies, and with its help the velocity distribution of the points of rigid bodies is carried out at a given time of their motion. With Euler formula a series of properties can be established of velocity distributions of the points of a solid rigid body found in a general motion.

Without demonstrating those, we shall present in the following, some of the most important properties of the velocity distribution in the general motion of the rigid body.

Vector $\bar{\omega}$ is the same in any point of the rigid body. Vector $\bar{\omega}$ does not depend on the choice of the origin of the mobile reference system that is, $\bar{\omega}$ is an invariant in relation to the axes solidary with the rigid body.

The projections of the velocities of two points in the solid body on the line uniting those, are equal and of the same sense: theorem of the velocity projections (Fig. 2).

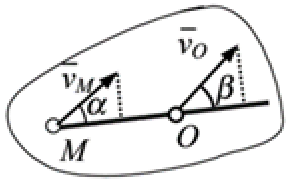

Fig. 2. Velocity Projections

The extremities of the velocity vector of certain collinear points in a solid body in a general motion, are in their turn collinear: theorem of collinearity of velocity vector extremities (Fig. 3).

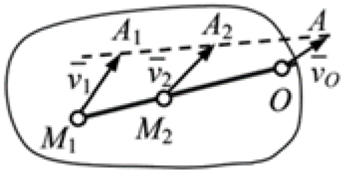

Fig. 3. Example of a Figure Caption (Figure Caption)

Velocity projections of various points of a solid rigid body found in a general motion, in the direction of vector are constant (Fig. 4). For this property the following comments are made[25][26].

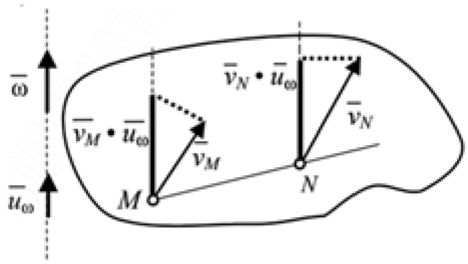

Fig. 4. Velocity Projections on the Line $\| \bar{\omega}$

This property shows that there are no null velocity points in the general movement of the rigid body. If the vectors and are perpendicular in a point, the property stays valid for all the points of the rigid body. The velocity distribution in the general motion of the rigid body, has a second invariant (scalar invariant), namely, projection of velocity $\bar{v}$ in the direction of the vector $\bar{\omega}$:

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$$v_{\omega} = \frac{\bar{v} \bullet \bar{\omega}}{|\bar{\omega}|} \quad (6)$$

The points in a rigid body found in a general motion and which are situated on a line parallel with the direction of the vector $\bar{\omega}$ have the same velocity (Fig. 5).

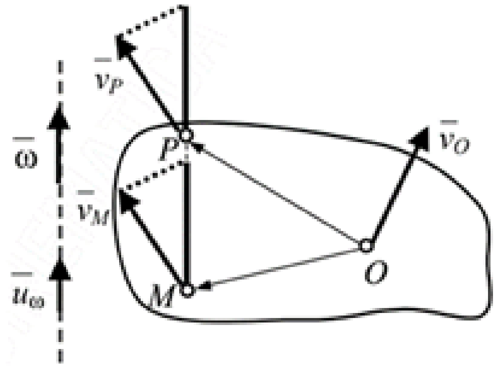

Fig. 5. Points with $\bar{v} \in (\Delta) \| \bar{\omega}$

The property is useful for the study of various particular movements, such as rotational movement, helical movement, etc.

B. Collinearity Property of the Extremities of the Velocity Vectors

The statement of the property (theorem) of collinearity of the extremities of the velocity vectors is: the extremities of the velocity vectors (drawn at the same scale) of three collinear points belonging to a rigid body in motion are collinear as well. Since a vector is characterized by size, direction and sense, having an origin (the point of application of the vector) and an extremity (the latter being generally noted with letters, they being considered as geometric points), the problem leads to demonstrating the collinearity of three points (the extremities of the velocity vectors). In geometry, collinearity is the property of a number greater than two points to belong to the same line. Several non-collinear points are points that cannot belong to the same line. This can be also demonstrated using vectors and complex numbers, as for coplanarity. Out of the methods specific to demonstrate collinearity used in geometry, we mention the following methods:

- Demonstration of collinearity with the help of elongated angle (additional angles)

If A and C are situated on either side of line BD, and m(<ABD)+m(<DBC)=180º, then points A, B and C are collinear (Fig. 6).

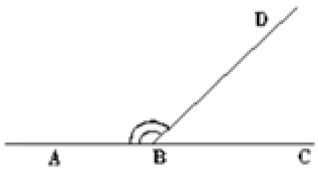

Fig. 6. Additional angles

Demonstration of collinearity using the reciprocal of the theorem of opposite angles at the apex.

If point B is situated on line DE, and A, and C are on either side of line DE, and <ABD = <CBE, then points A, B, C are collinear (Fig. 7).

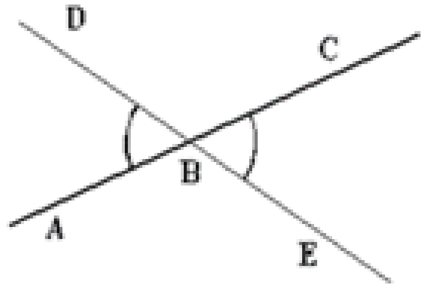

Fig. 7. Angles Opposed at the Peak

- Demonstration of collinearity by identifying a line that includes the respective points.

To show that points A, B, C are collinear, a line is identified to which they should belong.

- The condition of collinearity of three points, A(x1,y1), B(x2,y2), C(x3,y3) is obtained if we stipulate that C(x3,y3) point would verify the equation of line AB, that is:

$$\frac{y_3 - y_1}{y_2 - y_1} = \frac{x_3 - x_1}{x_2 - x_1} \quad (7)$$

The condition of collinearity of the three points can also be written in the form of a determinant:

$$\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} \quad (8)$$

III. PROBLEM SOLUTION

A. Properties of Velocity Distribution in Rigid Bodies

Two vectors are collinear if they have the same direction. This happens when both vectors are nonnull and their supporting lines are parallel or coincide, in the case when one of the vectors is null. The parallelism of the vectors represents a particular case of their collinearity, which is explicable by the fact that free vectors have no fixed position and can be translated in any point of the plane.

B. Demonstration of Collinearity of Points Using the Applications of Complex Numbers in Geometry

If points A, B, C have respectively zA, zB, zC, affixes, then A, B, C are collinear if and only if $(z_B - z_A)/(z_C - z_A) \in \mathbb{R}^*$.

In the following we intend to demonstrate the property of collinearity of velocity vectors of velocity distribution in the general movement of the rigid body, by approaching different methods of demonstrating collinearity.

Consider the points belonging to a rigid body in movement and collinear M_1, M_2, and M_3 (Fig. 8), which is written vectorially in the form:

$$\overline{M_1 M_2} = \lambda \overline{M_1 M_3} \quad (9)$$

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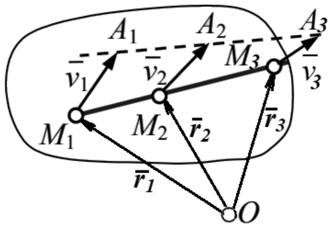

Fig. 8. Schema of Collinearity of Velocity Extremities

At the scale of the drawing, the velocity of points M_1, M_2 and M_3 are:

$$\begin{aligned} \bar{v}_1 &= k\overline{M_1A_1} \\ \bar{v}_2 &= k\overline{M_2A_2} \\ \bar{v}_3 &= k\overline{M_3A_3} \end{aligned} \tag{10}$$

The proportionality factor k is called the velocity scale. Equation (9), depending on the vectors of position is written in the form:

$$\bar{r}_2 - \bar{r}_1 = \lambda(\bar{r}_3 - \bar{r}_1) \tag{11}$$

Deriving the equation (11) in relation to time, considering that $\dot{\bar{r}}_1 = \bar{v}_1, \dot{\bar{r}}_2 = \bar{v}_2, \dot{\bar{r}}_3 = \bar{v}_3$ we get:

$$\bar{v}_2 - \bar{v}_1 = \lambda(\bar{v}_3 - \bar{v}_1) \tag{12}$$

At the scale of the drawing, (12) becomes:

$$\overline{M_2A_2} - \overline{M_1A_1} = \lambda\left(\overline{M_3A_3} - \overline{M_1A_1}\right) \tag{13}$$

Summing up element by element (11) and (13), the following results:

$$\begin{aligned} &\left(\bar{r}_2 + \overline{M_2A_2}\right) - \left(\bar{r}_1 + \overline{M_1A_1}\right) = \\ &\lambda\left[\left(\bar{r}_3 + \overline{M_3A_3}\right) - \left(\bar{r}_1 + \overline{M_1A_1}\right)\right] \end{aligned} \tag{14}$$

Or:

$$\overline{OA_2} - \overline{OA_1} = \lambda\left(\overline{OA_3} - \overline{OA_1}\right) \tag{15}$$

Consequently:

$$\overline{A_1A_2} = \lambda\overline{A_1A_3} \tag{16}$$

wich demonstrates both the collinearity of points A_1, A_2, A_3, and the fact that point A_2 divides segment A_1A_3 in the same ratio in which M2 divides segment M_1M_3.

The following demonstration of the collinearity theorem of the velocity extremities of three collinear point of a solid rigid body found in a general motion will be done using Euler's formula.

Consider collinear points O, M_1, M_2 of a solid body and points A, A_1, A_2 as being the extremities of velocity vectors $\bar{v}_0, \bar{v}_1, \bar{v}_2$ (Fig. 3). Considering point O as origin of the mobile system solidary with the rigid body, according to Euler's equation velocities $\bar{v}_1, \bar{v}_2$ have the following equations:

$$\bar{v}_1 = \bar{v}_0 + \bar{\omega}\times\overline{OM_1}, \quad \bar{v}_2 = \bar{v}_0 + \bar{\omega}\times\overline{OM_2} \tag{17}$$

Where: the position vectors $\overline{OM_1}$ and $\overline{OM_2}$ can be expressed as geometric sums (Fig. 3):

$$\begin{aligned} \overline{OM_1} &= \overline{OA} + \overline{AA_1} + \overline{A_1M} = \bar{v}_O + \overline{AA_1} - \bar{v}_1 \\ \overline{OM_2} &= \overline{OA} + \overline{AA_2} + \overline{A_2M} = \bar{v}_O + \overline{AA_2} - \bar{v}_2 \end{aligned} \tag{18}$$

Considering (17), the previous expressions (18) will get the form:

$$\begin{aligned} \overline{OM_1} &= \overline{AA_1} - \bar{\omega}\times\overline{OM_1} \\ \overline{OM_2} &= \overline{AA_2} - \bar{\omega}\times\overline{OM_2} \end{aligned} \tag{19}$$

If the three points O, M_1, M_2 are collinear, then:

$$\overline{OM_1} = \lambda\overline{OM_2} \tag{20}$$

From (19) substituting in (20) the following is inferred:

$$\overline{AA_1} - \bar{\omega}\times\overline{OM_1} = \lambda\left(\overline{AA_2} - \bar{\omega}\times\overline{OM_2}\right) \tag{21}$$

Or:

$$\overline{AA_1} = \lambda\cdot\overline{AA_2} - \bar{\omega}\times\left(\overline{OM_1} - \lambda\cdot\overline{OM_2}\right) \tag{22}$$

But: $\overline{OM_1} - \lambda\cdot\overline{OM_2}$ by virtue of (20). It results $\overline{AA_1} = \lambda\cdot\overline{AA_2}$, that is, the three points A, A_1, A_2 are collinear.

Another method to demonstrate collinearity is the one using vectorial product, knowing that it is annulled when the vectors of the products are collinear. If: $\overline{A_1A_2}\times\overline{A_1A_3} = 0$, then the two vectors are collinear, and the three points A_1, A_2, A_3 respectively, are also collinear. According to Fig. 9 the following equations result:

$$\begin{aligned} &\overline{M_1M_2} + \bar{v}_2 = \bar{v}_1 + \overline{A_1A_2} \\ &\Rightarrow \overline{A_1A_2} = \bar{v}_2 - \bar{v}_1 + \overline{M_1M_2} \\ &\overline{M_1M_3} + \bar{v}_3 = \bar{v}_1 + \overline{A_1A_3} \\ &\Rightarrow \overline{A_1A_3} = \bar{v}_3 - \bar{v}_1 + \overline{M_1M_3} \end{aligned} \tag{23}$$

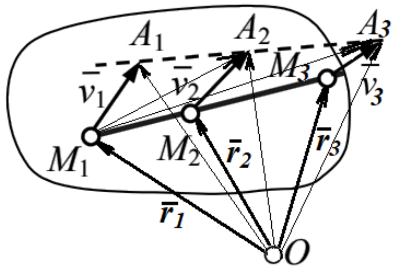

Fig. 9. Schema for Calculation with Vectorial Product

With (23) we have:

$$\overline{A_1A_2}\times\overline{A_1A_3} = \left(\bar{v}_2 - \bar{v}_1 + \overline{M_1M_2}\right)\times\left(\bar{v}_3 - \bar{v}_1 + \overline{M_1M_3}\right) \tag{24}$$

But: $\overline{M_1M_3} = \lambda\overline{M_1M_3} \Leftrightarrow \bar{r}_3 - \bar{r}_1 = \lambda(\bar{r}_2 - \bar{r}_1) \overset{(\,)'}{\Rightarrow} \bar{v}_3 - \bar{v}_1 = \lambda(\bar{v}_2 - \bar{v}_1)$

and (24) becomes:

$$\begin{aligned} &\overline{A_1A_2}\times\overline{A_1A_3} = \left(\bar{v}_2 - \bar{v}_1 + \overline{M_1M_2}\right)\times\left[\lambda(\bar{v}_3 - \bar{v}_1) + \lambda\overline{M_1M_2}\right] \\ &= \left(\bar{v}_2 - \bar{v}_1 + \overline{M_1M_2}\right)\times\lambda\left(\bar{v}_2 - \bar{v}_1 + \overline{M_1M_2}\right) \equiv 0 \end{aligned} \tag{25}$$

Thus, the three points A_1, A_2, A_3 are also collinear.

C. Collinearity Condition of three Points Moving on different Trajectories

- Applying the property of collinearity.

In the following we shall present an application in which the collinearity principle is used.

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The following problem is considered: in the same moment and from the same point O, three bodies are launched in a gravitational field, with different initiated velocities v_1, v_2, v_3 represented in Fig. 10.

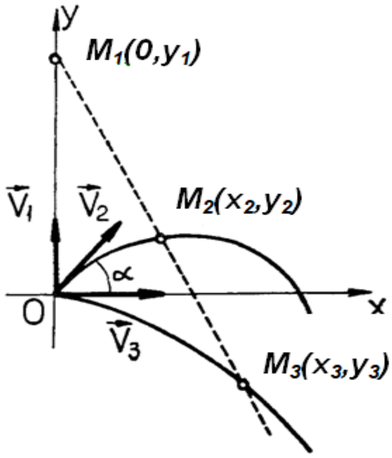

Fig. 10. Launching Points

We aim to find the relationship that should exist between the magnitudes of initial velocities and angle α, so that all along the motion, the three bodies would remain collinear. In order to solve such a collinearity problem, we should admit the following two simplifying hypotheses, which have not been included in its statement: ignoring the air resistance and considering the three bodies as material points. To establish the collinearity condition of the three material points, $M_1(0,y_1)$, $M_2(x_2,y_2)$ and $M_3(x_3,y_3)$, we appeal at first to our knowledge of analytical geometry. The collinearity condition lies in the following equation according to (8):

$$\delta = \begin{vmatrix} 0 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \tag{26}$$

To express the coordinates of points M_1, M_2 and M_3 at any time t > 0, calculated from the moment of launching the bodies, we appeal to our knowledge of kinematics from mechanics:

- vertical throwing up:

$$x_1 = 0, \quad y_1 = v_1 t - \frac{g}{2}t^2 \tag{27}$$

- obliquely throwing:

$$x_2 = v_2 t \cos\alpha$$
$$y_2 = v_2 t \sin\alpha - \frac{g}{2}t^2 \tag{28}$$

- horizontally throwing:

$$x_3 = v_3 t, \quad y_3 = -\frac{g}{2}t^2 \tag{29}$$

Substituting then (27), (28) and (29) in determinant (26) we get:

$$\delta = \begin{vmatrix} 0 & v_1 t - \frac{g}{2}t^2 & 1 \\ v_2 \cos\alpha & v_2 t \sin\alpha - \frac{g}{2}t^2 & 1 \\ v_3 t & -\frac{g}{2}t^2 & 1 \end{vmatrix} \tag{30}$$

so that the development of this determinant leads to the required condition:

$$v_3 t\left(v_1 t - \frac{g}{2}t^2\right) = \frac{g}{2}v_2 t^3 \cos\alpha - v_3 t\left(v_2 t \sin\alpha - \frac{g}{2}t^2\right)$$
$$- v_2 t \cos\alpha \cdot \left(v_1 t - \frac{g}{2}t^2\right) = 0 \tag{31}$$

After reducing the similar terms and dividing by t_2, (t > 0), we get in the end the equation searched:

$$v_2 v_3 \sin\alpha + v_1 v_2 \cos\alpha - v_1 v_3 = 0 \tag{32}$$

- Demonstration of collinearity of the points using vectors.

We shall further demonstrate the collinearity of the points using vectors this time. Vectors $\overline{r}_A = (x_A, y_A)$ and $\overline{r}_B = (x_B, y_B)$ are collinear if and only if their coordinates (projections on axes) are proportional, namely:

$$\frac{x_A}{x_B} = \frac{y_A}{y_B} \tag{33}$$

If: $x_B, y_B \neq 0$, points A, B, C are collinear if and only if vectors $\overline{AB}$ and $\overline{AC}$ are collinear that is, if and only if exists, so that:

$$\overline{AB} = \alpha \overline{AC} \tag{34}$$

But:

$$\overline{AB} = \alpha\overline{AC} \;\Leftrightarrow\; (x_B - x_A)\overline{i} + (y_B - y_A)\overline{j}$$
$$= (\alpha x_C - \alpha x_A)\overline{i} + (\alpha y_C - \alpha y_A)\overline{j}$$
$$\Leftrightarrow (x_B - x_A) = \alpha(x_C - x_A)$$
$$(y_B - y_A) = \alpha(y_C - y_A) \tag{35}$$
$$\Leftrightarrow \frac{x_B - x_A}{x_C - x_A} = \frac{y_B - y_A}{y_C - y_A}$$
$$with: x_C - x_A \neq 0,\; y_C - y_A$$

Thus, according to Fig. 11, for points M_1, M_2 and M_3 we have:

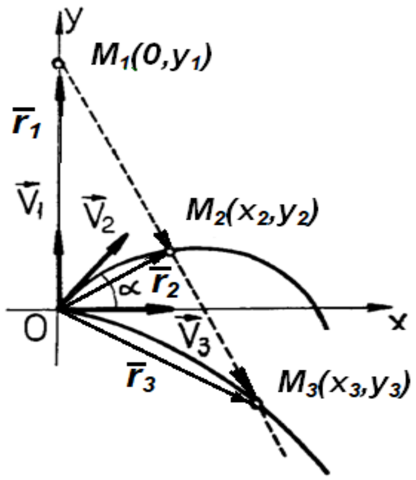

Fig. 11. Position Vectors of Points

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$$\bar{r}_1 = 0\bar{i} + y_1\bar{j},\ \ \bar{r}_1 = \left(v_t t - \frac{g}{2}t^2\right)\bar{j}$$

$$\bar{r}_2 = x_2\bar{i} + y_2\bar{j},\ \ \bar{r}_2 = v_2 t\cos\alpha\bar{i} + \left(v_2 t\sin\alpha - \frac{g}{2}t^2\right)\bar{j} \quad (36)$$

$$\bar{r}_3 = x_3\bar{i} + y_3\bar{j},\ \ \bar{r}_3 = v_3 t\bar{i} + \left(-\frac{g}{2}t^2\right)\bar{j}$$

For points M_1, M_2 and M_3 to be collinear the condition is:

$$\overline{M_1M_3} = \alpha\overline{M_1M_2} \quad or \quad \frac{x_3 - x_1}{x_2 - x_1} = \frac{y_3 - y_1}{y_2 - y_1} \quad (37)$$

Thus, it results:

$$\frac{v_3 t}{v_2 t\cos\alpha} = \frac{v_1 t}{-(v_2 t\sin\alpha - v_1 t)} \Rightarrow \ -v_3 v_2\sin\alpha + v_3 v_1 = v_1 v_2\cos\alpha \quad (38)$$

It is noticed that (37) is identical to (31).

- Determining the collinearity relationship of three points using complex numbers.

Next, we shall determine the collinearity equation for points M_1, M_2 and M_3 using complex numbers. Associating z = x + iy, M(x, y) to set R of real numbers, Ox axis corresponds, called in this context, the real axis, and to set *iR* of imaginary numbers, axis Oy, called imaginary axis. The plane the points of which are identified with complex numbers by function g o f, previously defined, is called complex plane. Affixes of points M_1, M_2 and M_3 (fig. 12) are z_1, z_2, z_3. Points $M_1(z_1)$, $M_2(z_2)$ and $M_3(z_3)$ are collinear if and only if $(z_3-z_1)/(z_3-z_2)$ belongs to R.

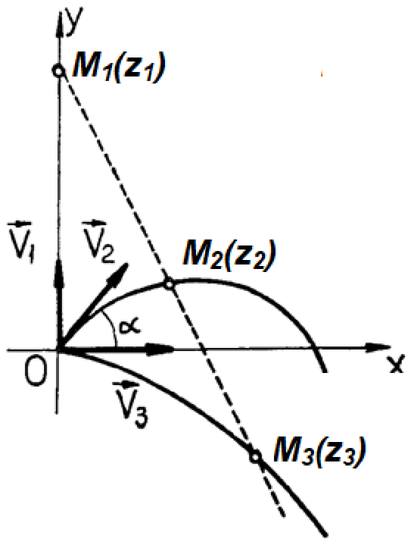

Fig. 12. Affixes of Points M_1, M_2 and M_3

According to Fig. 12 and the kinematic equations from mechanics, we have:

$$z_1 = x_1 + iy_1,\quad z_1 = i\left(v_1 t - \frac{g}{2}t^2\right)$$

$$z_2 = x_2 + iy_2,\ z_2 = v_2 t\cos\alpha + i\left(v_2 t\sin\alpha - \frac{g}{2}t^2\right) \quad (39)$$

$$z_3 = x_3 + iy_3,\quad z_3 = v_3 t + i\left(-\frac{g}{2}t^2\right)$$

$$\frac{z_3 - z_1}{z_2 - z_1} = \frac{x_3 + iy_3 - (x_1 + iy_1)}{x_2 + iy_2 - (x_1 + iy_1)} = \frac{x_3 - x_1 + i(y_3 - y_1)}{x_2 - x_1 + i(y_2 - y_1)}$$

$$\Leftrightarrow \frac{v_3 t - 0 + i\left(-\frac{g}{2}t^2 - v_1 t + \frac{g}{2}t^2\right)}{v_2 t\cos\alpha - 0 + i\left(v_2 t\sin\alpha - \frac{g}{2}t^2 - v_1 t + \frac{g}{2}t^2\right)} \quad (40)$$

$$= \frac{v_3 t - iv_1 t}{v_2 t\cos\alpha + i(v_2 t\sin\alpha - v_1 t)} = \lambda$$

Where: $\lambda \in \mathbb{R}^*$.

Thus, we shall have:

$$\frac{v_3 t - iv_1 t}{v_2 t\cos\alpha + i(v_2 t\sin\alpha - v_1 t)} = \lambda$$

$$\left\{v_3 t - iv_1 t = \lambda\left[v_2 t\cos\alpha + i(v_2 t\sin\alpha - v_1 t)\right]\right\}\frac{1}{t} \quad (41)$$

$$v_3 - \lambda v_2\cos\alpha = i\left[v_1 + \lambda(v_2\sin\alpha - v_1)\right]$$

$$v_3 - \lambda v_2\cos\alpha - i\left[v_1 + \lambda(v_2\sin\alpha - v_1)\right] = 0$$

For the last equation of (40) to be $v_3 - \lambda v_2\cos\alpha - i\left[v_1 + \lambda(v_2\sin\alpha - v_1)\right] = 0$ we must have: $v_3 - \lambda v_2\cos\alpha = 0$ and $v_1 + \lambda(v_2\sin\alpha - v_1) = 0$, whence:

$$\lambda = \frac{v_3}{v_2\cos\alpha} \quad and \quad \lambda = \frac{v_1}{v_1 - v_2\sin\alpha} \quad (42)$$

Equalizing the two equations of (42), it results:

$$\frac{v_3}{v_2\cos\alpha} = \frac{v_1}{v_1 - v_2\sin\alpha} \Rightarrow \ v_3(v_1 - v_2\sin\alpha) = v_1 v_2\cos\alpha \quad (43)$$

$$\Rightarrow \ v_2 v_3\sin\alpha + v_1 v_2\cos\alpha - v_1 v_3 = 0$$

Equation (42) is identical with (32).

- Determination of collinearity by means of the elongated angle.

We shall further consider the demonstration of collinearity with the help of the elongated angle (additional angles). If M_1 and M_3 are situated on one side and the other of line OM2 and m(<M_1M_2O) + m(<OM_2M_3) = 180° (Fig. 13), then points M_1, M_2 and M_3 are collinear.

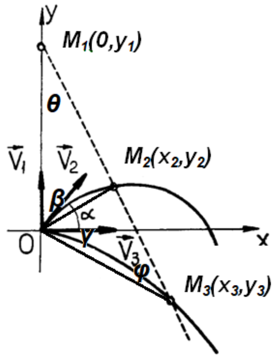

Fig. 13. Case of Elongated Angle

Taking into consideration Fig. 13, we shall make the following notations:

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$$\begin{aligned}&\measuredangle OM_1M_2=\theta, \quad \measuredangle M_1OM_2=\beta\\ &\measuredangle OM_3M_2=\varphi, \quad \measuredangle M_2OM_3=\gamma\end{aligned} \tag{44}$$

From ΔOM_1M_2 results that:

$$\measuredangle OM_2M_1=180^0-(\beta+\theta) \tag{45}$$

From ΔOM_3M_2 results that:

$$\measuredangle OM_2M_3=180^0-(\gamma+\varphi) \tag{46}$$

Calculating the sum of the extent of angles $\measuredangle OM_2M_1$ and $\measuredangle OM_2M_3$, we get:

$$\begin{aligned}&\measuredangle OM_2M_1+\measuredangle OM_2M_3=180^0-(\gamma+\varphi)+180^0\\ &-(\beta+\theta)=360^0-(\gamma+\varphi+\beta+\theta)\end{aligned} \tag{47}$$

But:

$$(\gamma+\varphi+\beta+\theta)=180^0 \tag{48}$$

Equation (47) becomes:

$$\measuredangle OM_2M_1+\measuredangle OM_2M_3=180^0 \tag{49}$$

Whence it results that points M_1, M_2 and M_3 are collinear and then the vectors $\overline{M_1M_2}$ and $\overline{M_1M_3}$ being collinear, their vectorial product is null. Thus, we have:

$$\begin{aligned}&\overline{M_1M_2}\times\overline{M_1M_3}\\ &=\left[v_2t\cos\alpha\,\bar{i}-(v_2t\sin\alpha-v_1t)\,\bar{j}\right]\times\left(v_3t\bar{i}+v_1t\bar{j}\right)\\ &=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ v_2t\cos\alpha & v_1t-v_2t\sin\alpha & 0\\ v_3t & v_1t & 0\end{vmatrix}=0\end{aligned} \tag{50}$$

From the development of the determinant, we get:

$$v_2v_3\sin\alpha+v_1v_2\cos\alpha-v_1v_3=0 \tag{51}$$

Equation (51) being identical with equation (32). Such an issue lends itself to interesting discussions. Obviously, collinearity is not possible if condition (32) is not met, and which, as it is noticed, has a certain symmetry. Moreover, it is noticed that the disposition of the velocities of the three bodies is symmetrical in relation to axis Oy, the result (32) remaining. In this context, other dispositions of velocities of the three bodies can be looked for, in order to meet the condition of collinearity, if not all along the motion, at least for certain moments considered in relation to the moment of simultaneous launching of the bodies.

Next, we shall limit ourselves only to the condition (32), assuming v_1, v_2, v_3 being given, and trying to determine α, for which collinearity is maintained. To this end, we must solve equation (32) in relation to the unknown α.

By working out this equation, we shall express $\sin\alpha$ and $\cos\alpha$ by $x=tg(\alpha/2)$:

$$\sin\alpha=\frac{2x}{1+x^2}, \quad \cos\alpha=\frac{1-x^2}{1+x^2} \tag{52}$$

Substituting (52) in (32) we get the algebraic equation of the form:

$$v_1(v_2+v_3)x^2+2v_2v_3x-v_1(v_2-v_3)=0 \tag{53}$$

Equation (53) has real solutions insofar as its discriminant $\Delta\geq 0$, that is, insofar as:

$$v_2^2v_3^2+v_1^2\left(v_2^2-v_3^2\right)\geq 0 \tag{54}$$

In this case, the solutions for (53) are:

$$x_{1,2}=\frac{-v_2v_3\pm\sqrt{v_2^2v_3^2+v_1^2\left(v_2^2-v_3^2\right)}}{v_1(v_2+v_3)} \tag{55}$$

Remaining only in the first quadrant of the system xOy, $\alpha\in(0,\pi/2)$, the only possible solution for x is:

$$x=x_1=\frac{\sqrt{v_2^2v_3^2+v_1^2\left(v_2^2-v_3^2\right)}-v_2v_3}{v_1(v_2+v_3)} \tag{56}$$

Obviously, the solution (56) is acceptable if $v_2 > v_3$ and, as a result:

$$\begin{aligned}&\alpha=2arctgx\\ &\alpha=2arctg\frac{\sqrt{v_2^2v_3^2+v_1^2\left(v_2^2-v_3^2\right)}-v_2v_3}{v_1(v_2+v_3)}\end{aligned} \tag{57}$$

IV. CONCLUSION

Interdisciplinarity is a cooperation between various disciplines of the same curricular area, regarding a certain phenomenon, process, the complexity of which can be demonstrated, explained, solved, only by the action of several factors. Interdisciplinarity involves approaching the complex contents with the aim of forming a unitary image on a certain subject matter. This implies combining two or several academic disciplines in one single activity. Thus, new knowledge is accumulated in several fields simultaneously. Mechanics depends on mathematics, and we can realize this by the fact that we cannot solve any problem of mechanics without mathematics.

In the paper the tight connection between the two fundamental disciplines has been shown, mathematics and mechanics. For the time being, stopping here with the discussion of the problem, we should mention that such a discussion is far from exhausting the multitude of aspects that can be raised. This is a small example supporting the principle of continuity of knowledge, of the fact that the process of knowledge is unlimited, and as in any science, it stays open all the time for acquiring new contributions.

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