Study of the Performance of 3x3 MIMO Transmission System using MMSE and ML Detectors

Pramodini DV, AG Ananth, HM Mahesh

***Abstract:** Analysis of full-rate linear and space-time block code for 3X3 multiple-input multiple-output (MIMO) communication systems under a Rayleigh flat-fading environment has been carried out. The design targets specifically use a Linear Minimum Mean-square Error (MMSE) and Maximum Likelihood (ML) receivers which minimizes the average Symbol error rate (SER) for a QPSK transmitted signal. The optimization problem is solved by minimizing a lower bound of the SER. The performance of MIMO systems has been studied by changing the number of transmitters and receivers. A comparison has been made between MMSE and ML receivers in terms SER as a function of SNR. The simulation results show that the higher MIMO system exhibits a SNR~4 dB improvement for ML receiver in comparison to linear MMSE receiver.*

***Index Terms:** MIMO, MMSE, ML, QPSK, SER, SNR.*

I. INTRODUCTION

Linear receivers are the complex spatial multiplexing receivers. There are two types of linear receivers zero forcing (ZFR) and minimum mean square estimator (MMSE). The MMSE receiver gives slightly better performance as compared to ZFR which is discussed in the previous paper. However, with a new optimal ordering method in V-BLAST, performance can be improved in terms handling bits by exploiting the whole filter output [1], which is used in the current work. A suboptimal ordering metric is also proposed which requires much reduced complexity compared to the optimal ordering metric. A simplified version of the suboptimal ordering metric, which achieves a significant performance gain over the conventional ordering with minor additional complexity is derived.

It is also considered that the design of a full-rate linear space-time block code for coherent MIMO communication systems under a Rayleigh flat fading environment. The design targets specifically at the use of a linear MMSE receiver [6] that minimizes the asymptotic average SER when the transmitted signal is selected from a 4-QAM constellation. This optimization problem is solved by minimizing a lower bound on the SER. By exploiting an optimization technique without any assumption on the code, we prove that individual unitary and trace-orthogonal structures [2] are the necessary and sufficient conditions to assure the minimum asymptotic average SER with an MMSE detector. An algorithm is provided for an efficient generation of our codes. Later ML equalizer is used to improve SER [7].

The multiplexing gain of multiple antenna transmission strongly depends on transmit and receive antenna spacing, transmit antenna synchronization, and the algorithm used to eliminate inter-channel interference (ICI) at the receiver [3]. In transmission approach, called spatial modulation, that entirely avoids ICI and requires no synchronization between the transmitting antennas while maintaining high spectral efficiency. A block of information bits is mapped into a constellation point in the signal and the spatial domain, i.e. into the location of a particular antenna [4].

The receiver estimates the transmitted signal and the transmit antenna number and uses the two information to de-map the block of information bits. For this purpose, the reference of transmit diversity detection maximum ratio combining (MRC) is considered. Spatial modulation is used to transmit different number of information bits and MRC is used to estimate both the transmitted signal and the transmit antenna number. The SER performance and the achieved spectral efficiency are comparable to V-BLAST [5]. However, spatial modulation results in a vast reduction in receiver complexity.

II. IMPLEMENTATION

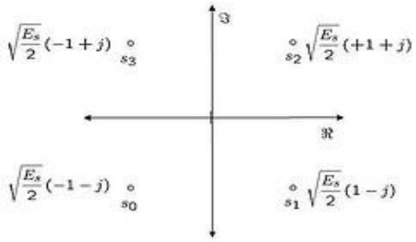

Fig 1. Constellation plot for QPSK

The scaling factor of √Es/2 is for normalizing the average energy of the transmitted symbols to 1, assuming that all the constellation points are equally likely. In the noise model, assume that the additive noise n follows the Gaussian probability distribution function,

Manuscript published on 30 October 2011.

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$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$ With $\mu = 0$ and $\sigma 2 = \frac{N_0}{2}$.

Probability of error computation:

Consider the symbol **s_2.** The conditional probability distribution function (PDF) of y given $s_2$was transmitted is:

$P\,(y|S_2) = 1/(\sqrt{\Pi}N_0)\, e^{-(y-\sqrt{Es/2})2\,/\,N_0}$

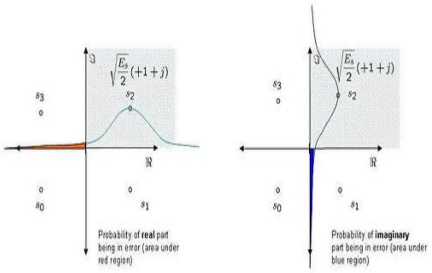


Fig 2: Probability density function for QPSK

From the figure 2, the symbol s_2 is decoded correctly only if y falls in the area in the hashed region i.e. $P\,(c|S_2) = p\,(R_y>0|s_2)\; p\,(I_y>0|s_2)$. Probability of real component of y greater than 0, given s_2 was transmitted is (i.e. area outside the red region) $P\;(R_y>0|s_2) = 1-1/\;(\sqrt{\pi}N_0)((\int e\text{-}(R_y-\sqrt{E_s/2})/N_0)^2)\;dy = 1-1/2\mathrm{erfc}(\sqrt{E_s/2N_0})$ at the integration y to ∞ , where the complementary error function, $\mathrm{erf_c}(x) = 2/\sqrt{\Pi}\;\int e^{-x2}\,dx$ at the integration x to ∞.

Similarly, probability of imaginary component of y greater than 0, given $s_2$was transmitted is (i.e. area outside the blue region).

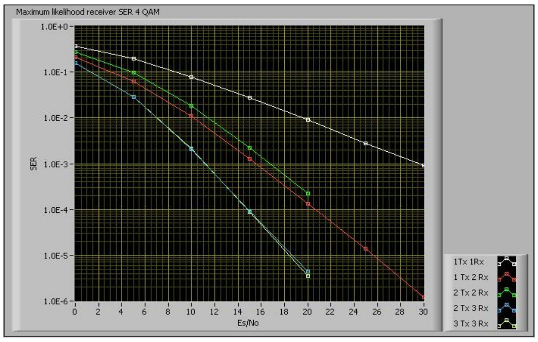
$P(I_y>0|s_2) = 1-1/(\sqrt{\pi}N_0)\;((\int\; e\text{-}(Iy-\sqrt{E_s/2})/N_0)^2)\;dy = 1-1/2\mathrm{erfc}(\sqrt{E_s/2N_0})$. The probability of $s_2$being decoded correctly is, $P(c|S_2) = 1-\;1/2\mathrm{erfc}(\sqrt{E_s/N_0})^2$

$= 1-\;2/2\mathrm{erfc}\;(\sqrt{E_{s/2}N_0}) + 1/2\mathrm{erfc}^2\,(\sqrt{E_s/2N_0}) = 1-\;\mathrm{erfc}(\sqrt{E_s/2N_0}) + 1/4\mathrm{erfc}^2(\sqrt{E_s/2N_0})$

Total symbol error probability in which the symbol will be in error, if atleast one of the symbols is decoded incorrectly. The probability of symbol error is, $P_{QPSK} = 1-\;P\;(c|S_2) = 1-\;1-\mathrm{erfc}(\sqrt{E_s/2N_0}) + 1/4\mathrm{erfc}^2(\sqrt{E_s/2N_0})$

For higher values of $E_s/2$, the second term in the equation becomes negligible and the probability of error can be approximated as, $P_{QPSK} \sim \mathrm{erf_c}\;(\sqrt{E_s/2N_0})$

III. SIMULATION RESULTS AND DISCUSSIONS



The simulation studies have been carried out using the Lab View software. The SER values have been computed as a function of SNR for different 2x2, 2x3, 2x4, and 3x3 MIMO systems using QPSK transmission for both ML and MMSE receivers. The figure 3 shows the SER v/s SNR performance for different MIMO systems with ML receiver. The figure 2 shows that for SER at ~10^{-3} point in comparison to 2x2 MIMO systems, the 2x3 MIMO shows an improvement in SNR ~6 dB. Similarly for 2x4 MIMO system SNR ~ 4dB and for 3x3 MIMO systems SNR ~ 5dB has been observed respectively for ML receiver configuration.

Fig 3. SER v/s SNR plot for MIMO with ML equalization for QPSK in Rayleigh channel.

Similarly, figure 4 shows SER v/s SNR plot for MMSE equalizer detector for various MIMO systems with QPSK modulation transmission. The figure shows the SER v/s SNR performance plotted for different MIMO systems with linear MMSE receiver. The figure shows that at SER at 10^{-3} point, in comparison to 2x2 MIMO system, the 2x3 MIMO shows an improvement in SER ~6 dB, for 2x4 MIMO system SER ~4dB and 3x3 MIMO system SER ~5dB respectively for MMSE equalizer detector configuration.

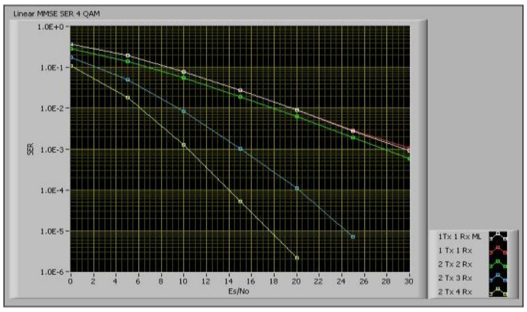


Fig 4. SER v/s SNR plot for MIMO with Linear MMSE equalization for QPSK in Rayleigh channel.

Figure 5 shows a SER plot for V-BLAST MMSE receiver performance using QPSK. Here, the SER performance for different SER values for a 2X2, 2X3 and 2X4 MIMO systems have been plotted. At SER, values are ~20dB, ~ 12dB, and ~8dB respectively at 10^{-3} point. The results show there is an improvement observed in SNR of ~4dB and with linear MMSE equalizer ~3dB compared to that of ML equalizer.

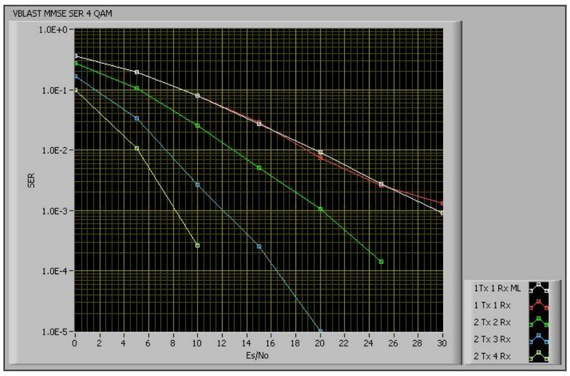


Fig 5. SER v/s SNR plot for MIMO with V-BLAST MMSE equalization for QPSK in Rayleigh channel.

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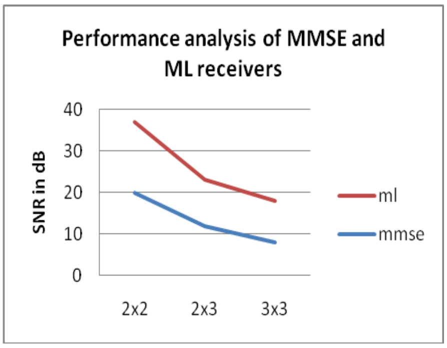

Fig 6. Performance analysis of MMSE and ML equalizers in terms of SNR v/s different MIMO systems.

Figure 6 shows a performance analysis graph in which ML and MMSE equalizers were compared on the basis of SNR in dB v/s different types of 2x2, 2x3 and 3x3 MIMO systems. It is very evident from the figure that the higher MIMO systems the ML receivers show a better performance in SNR compared to that of MMSE receivers.

IV. CONCLUSIONS

From the simulated results and discussions presented above it can be concluded that:

1. For ML receiver configuration at SER at 10^{-3} point 2x3 MIMO shows an improvement in SNR ~6 dB, 2x4 MIMO system SNR ~4dB and 3x3 MIMO systems SNR ~5dB respectively.
2. For MMSE equalizer detector configuration, at SER 10^{-3} point, the 2x3 MIMO shows an improvement in SER ~6 dB, 2x4 MIMO system SNR ~4dB and 3x3 MIMO system SNR ~5dB respectively.
3. For V-BLAST MMSE equalizer the SER values at 10^{-3} point, for higher MIMO system there is an improvement observed in SNR of ~4dB and ~3dB compared to that of ML equalizer.
4. For higher MIMO systems, the MMSE detector shows a better SER performance compared to ML equalizer detector.

ACKNOWLEDGEMENT

The authors are thankful to the HOD, Department of Information Science and Engineering, PES School of Engineering and Sri. Hari Prasad, Director of Cognitive Center, RV College of Engineering for their constant encouragement in carrying this research work.

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