International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-3 Issue-4, April 2014

Noise Analysis of Common-Collector Amplifier using Stochastic Differential Equation

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Abstract— ***In this paper, we analyse the effect of noise in a common-collector amplifier working at high frequencies. Extrinsic noise is analyzed using time domain method employing techniques from stochastic calculus. Stochastic differential equations are used to obtain autocorrelation functions of the output noise voltage and other solution statistics like mean and variance. The analysis leads to important design implications for improved noise characteristics of the common-collector amplifier.***

Index Terms— ***common-collector amplifier, noise, stochastic differential equation, mean and variance.***

I. INTRODUCTION

The common-collector amplifier is the most widely used in analog circuit design. In this paper, we shall concentrate on the noise analysis of a common-collector amplifier. We analyze the effect of the noise signal on the output voltage. Noise can enter the circuit via various paths such as the noise from within the amplifier(intrinsic) and the noise signal which is fed externally(extrinsic).

Circuit noise analysis is traditionally done in frequency domain. The approach is effective in cases where the circuit is linear and time invariant. But the approach is not applicable for the extrinsic noise because the system may not be either linear or time invariant due to the switching nature of the signal picked [2]. In this paper we do analysis of extrinsic noise for the common-collector amplifier as shown in Fig.1.

For the stochastic model being used in this paper, the external noise is assumed to be a white Gaussian noise process. Although the assumption of a white Gaussian noise is an idealization, it may be justified because of the existence of many random input effects. According to the Central Limit Theorem, when the uncertainty is due to additive effects of many random factors, the probability distribution of such random variables is Gaussian. It may be difficult to isolate and model each factor that produces uncertainty in the circuit analysis. Therefore, the noise sources are assumed to be white with a flat power spectral density(PSD).

In this method, we shall follow a time domain approach based on solving a SDE. The method of SDEs in circuit noise analysis was used in [3] from a circuit simulation point of view. Their approach is based on linearization of SDEs about its simulated deterministic trajectory. In this paper we will use a different approach from which analytical solution to the SDE will be obtained. The analytical solution will take into account the circuit time varying nature and it will be shown that the noise becomes significant at high input signal frequencies. The main aim of our analysis is to observe the effect of noise present in the input signal on the output of the common-collector amplifier.

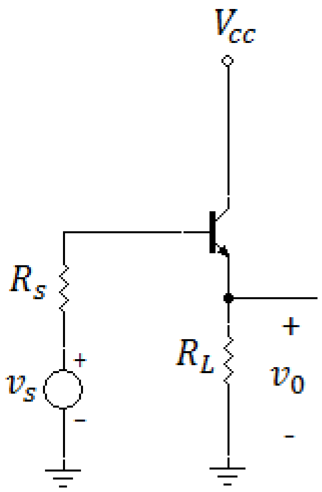

Fig.1. Common-Collector Amplifier

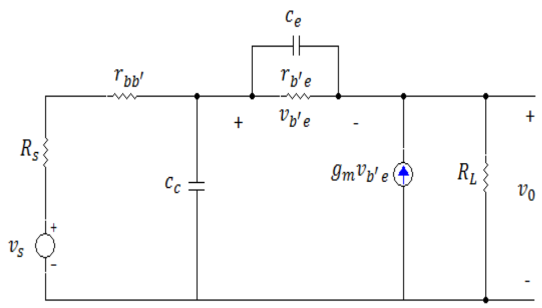

Fig.2. High-Frequency Equivalent Circuit

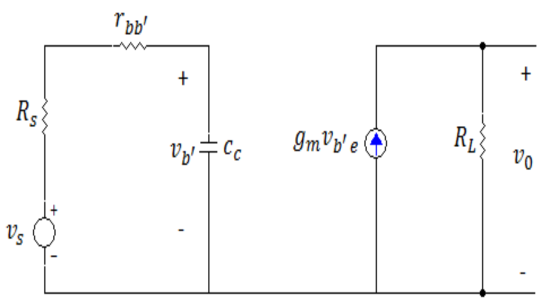

Fig.3. Simplify High-Frequency Equivalent Circuit

Manuscript published on 30 April 2014.
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II. ANALYSIS OF NOISE VIA SDEs

Consider a common-collector amplifier as shown Fig.1 whose high-frequency equivalent is Fig.2. Using Miller's theorem, we can transfer c_e and $g_{b'e}$ into input side by $c_e(1-k)$ and $g_{b'e}(1-k)$ and into output side by $c_e(1-k)/k$ and $g_{b'e}(1-k)/k$. Where $g_{b'e} = 1/r_{b'e}$. The gain (k) of the common-collector amplifier is close to 1, so the value of $1-k$ is approximately zero. So using this approximation, we can obtain the simplified high-frequency equivalent circuit, which is shown in Fig.3. Henceforth, we analyze the circuit using SDEs. From the circuit in Fig. 3,

$$\frac{v_s(t)-v_{b'}(t)}{R_{s'}} = c_c\frac{dv_{b'}(t)}{dt} \qquad (1)$$

where $R_s' = R_s + r_{bb'}$. Using some straightforward simplification (1) can be written as

$$\frac{dv_{b'}(t)}{dt} + k_1 v_{b'}(t) = \frac{v_s(t)}{c_c R_{s'}} \qquad (2)$$

where $k_1 = \frac{1}{c_c R_{s'}}$ and

$$v_0(t) = v_e(t) = g_m R_L v_{b'e}(t)$$

so $$v_0(t) = v_e(t) = \frac{g_m R_L}{1+g_m R_L} v_{b'}(t) \qquad (3)$$

Considering $v_s(t) = \sigma n(t)$, where $n(t)$ represents Gaussian noise process and σ^2 is the magnitude of PSD of input noise process. Substituting $v_s(t) = \sigma n(t)$ in (2), we obtain

$$\frac{dv_{b'}(t)}{dt} + k_1 v_{b'}(t) = \frac{\sigma n(t)}{c_c R_{s'}} \qquad (4)$$

First, we multiply both side of (4) with dt, then take expectation both sides. Since the continuous-time white noise process is a generalised function, the solution is rewritten by the replacement $n(t)\,dt = dW(t)$, where $W(t)$ is Wiener motion process, a continuous, but not differentiable process [4] .

$$dE[v_{b'}(t)] + k_1 E[v_{b'}(t)]dt = \frac{E[\sigma dW(t)]}{c_c R_{s'}} \qquad (5)$$

Using the fact that $E[\sigma dW(t)] = 0$, (5) results in the following:

$$\frac{dE[v_{b'}(t)]}{dt} + k_1 E[v_{b'}(t)] = 0 \qquad (6)$$

The solution of (6) is found out to be

$$E[v_{b'}(t)] = c_1 e^{-k_1 t} \qquad (7)$$

where c_1 is a constant whose value depends on the initial circuit conditions. From (3) and (7) we get the mean of the output

$$E[v_0(t)] = E[v_e(t)] = \frac{g_m R_L}{1+g_m R_L} E[v_{b'}(t)]$$

so $$E[v_0(t)] = \frac{g_m R_L}{1+g_m R_L} c_1 e^{-k_1 t} \qquad (8)$$

Next we find the autocorrelation function which will lead us to finding the variance. For the pedagogical reasons, the autocorrelation function is obtained considering initial conditions zero . Rewriting equation (2)

$$\frac{dv_{b'}(t)}{dt} + k_1 v_{b'}(t) = \frac{v_s(t)}{c_c R_{s'}} \qquad (9)$$

Next, we consider (9) at time $t = t_1$ with initial conditions $R_{v_{b'},v_{b'}}(0,t_2) = E[v_{b'}(t_1)v_{b'}(t_2)]|_{t_1=0} = 0$. Multiplying both sides of (9) with $v_{b'}(t_2)$ and then taking the expectation, we obtain

$$\frac{dR_{v_{b'},v_{b'}}(t_1,t_2)}{dt_1} + k_1 R_{v_{b'},v_{b'}}(t_1,t_2) = \frac{R_{v_s,v_{b'}}(t_1,t_2)}{c_c R_{s'}} \qquad (10)$$

Again, we consider (9) at time $t = t_2$ with initial conditions $R_{v_s,v_{b'}}(t_1,0) = E[v_s(t_1)v_{b'}(t_2)]|_{t_2=0} = 0$. Multiplying both sides of (9) with $v_s(t_1)$ and then taking the expectation, we obtain

$$\frac{dR_{v_s,v_{b'}}(t_1,t_2)}{dt_2} + k_1 R_{v_s,v_{b'}}(t_1,t_2) = \frac{R_{v_s,v_s}(t_1,t_2)}{c_c R_{s'}} \qquad (11)$$

Knowing that $R_{v_s,v_s}(t_1,t_2) = \sigma^2\delta(t_1-t_2)$, we find the solution of (11) as

$$R_{v_s,v_{b'}}(t_1,t_2) = \frac{\sigma^2}{c_c R_{s'}} e^{k_1(t_1-t_2)} \qquad (12)$$

Substituting the value of $R_{v_s,v_{b'}}(t_1,t_2)$ from (12) in (10) and taking the limit of t_1 from 0 to min(t_1,t_2), we obtain the solution of (10) as

$$R_{v_{b'},v_{b'}}(t_1,t_2) = \frac{\sigma^2}{2k_1(c_c R_s')^2}\left(e^{-k_1(t_1-t_2)} - e^{-k_1(t_1+t_2)}\right) \qquad (13)$$

For $t_1 = t_2 = t$ in (13) we obtain the second moment of $v_{b'e}(t)$ as $E[v_{b'e}^2(t)]$

$$E[v_{b'}^2(t)] = \frac{\sigma^2}{2k_1(c_c R_s')^2}(1-e^{-2k_1 t}) \qquad (14)$$

From (3) and (14) we obtain the second moment of output as $E[v_0^2(t)]$ (which is variance in this case)

$$E[v_0^2(t)] = E[v_e^2(t)] = \frac{(g_m R_L)^2}{(1+g_m R_L)^2} E[v_{b'}^2(t)]$$

so $$E[v_0^2(t)] = \frac{(g_m R_L)^2\sigma^2}{(1+g_m R_L)^2 2k_1(c_c R_s')^2}(1-e^{-2k_1 t}) \qquad (15)$$

III. SIMULATION RESULTS

For the simulation of the result obtain above, we use the following values for the circuit parameters $R_L = 10^4\Omega$, $R_s = 5\times10^3\Omega$, $r_{bb'} = 100\Omega$, $c_c = 0.8pF$, $\sigma = 0.25$, $g_m = 40mA/V$.

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The variation of mean with time is shown in Fig. 4, when initial conditions are nonzero, $(v_{b'}(0) = 0.01V)$. If initial conditions are zero the mean is zero all the time. The variation of variance with time is shown in Fig. 5. Initially the variance increases linearly with time then become constant.

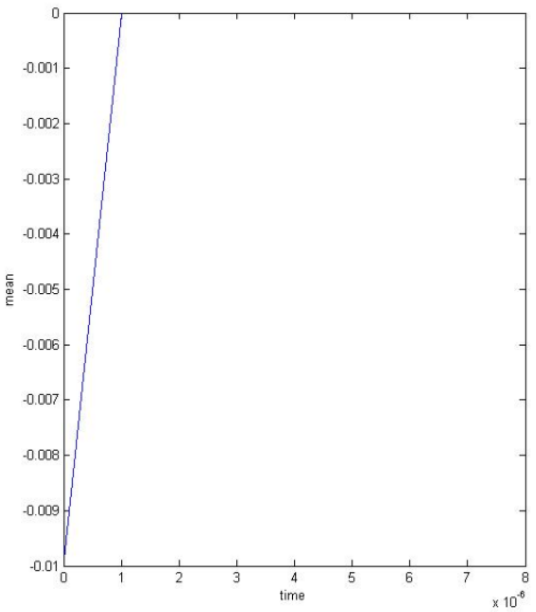

Fig.4. Variation of mean with time

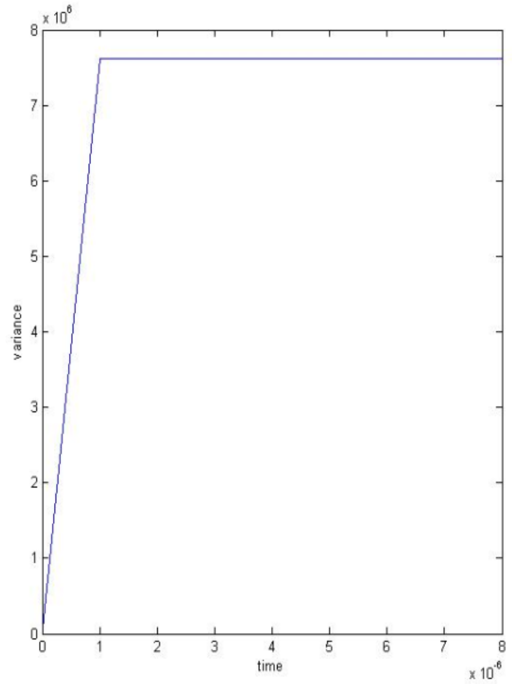

Fig.5. Variation of variance with time

IV. CONCLUSIONS

Noise in common-collector amplifier is analyzed using stochastic differential equation. Extrinsic noise is characterized by solving a SDE analytically in time domain. The solution for various solution statistics like mean and variance is obtained which can be used for design process. Suitable design methods which involve changing of device parameters are suggested to aid noise reduction and hence design the amplifier with reduced noise characteristics.

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