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Comparative Analysis of Two Different Adjustment Techniques as Applied to The Least Squares Processing of The Direct Linear Transformation (DLT)

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Abstract— A simple method for close range photogrammetric data reduction with non- metric cameras developed by Abdel Aziz and Karara [ ], it establishes the direct linear transformation (DLT) between coordinates of image points, and the corresponding object- space coordinates. This paper concentrates on the method of least squares as a method for adjusting the non-linear transformation equations. Hence, there are two different adjustment techniques that can be used to the processing of the non-linear transformation equations; namely: mixed and parametric least square adjustment techniques. The two different adjustment techniques are applied for two different geometry; namely double station geometry and multi-station geometry. In this study, two different iterative least – squares adjustment techniques are developed to solve the nonlinear transformation equations incorporated with distortion corrections. The results from the two different adjustment techniques will be discussed and analyzed. The obtained results showed the superiority of the mixed adjustment technique especially in the multi- station geometry. Therefore, the least squares mixed adjustment technique is recommended to be used for solving the non-linear transformations incorporated with distortion corrections.

Index Terms: *About four key words or phrases in alphabetical order, separated by commas.*

I. INTRODUCTION

Abdel Aziz and Karara (1971) developed the direct linear transformation (DLT) as a simple method for close – range photogrammetric data reduction with non-metric cameras. This method establishes the direct linear transformation (DLT) between the two-dimensional image coordinates and the three-dimensional object space coordinates. An insight in the different literature about using the different adjustment techniques in both geodesy and photogrammetry, one find that, the mixed adjustment technique has been utilized quite frequently in the geodetic applications, while in most of the photogrammetric applications, both parametric and condition adjustment techniques as simple special case of the mixed technique, are used instead. (Hirvonen (1971)) In this study, it has been found that either the mixed or the parametric adjustment technique can be used to solve the non-linear transformation equations. Consequently, only these two techniques of adjustment will be considered here. In this paper, two algorithms using an iterative least-squares adjustment scheme have been developed to solve the non-linear transformation equations incorporated with distortion corrections. According to the above discussion, the main objective of this research is to compare between the mixed and the parametric least-squares adjustment techniques, as applied to the least squares processing of a set of direct linear transformations. An overview of both mixed and parametric adjustment techniques will be reviewed. A simulated case study will be solved. The obtained results will be presented, discussed and the essential conclusions will be given.

II. MATHEMATICAL FORMULATION

Abdel-Aziz and Karara (1971), and numerous other authors (compare Karara,1979) have dealt with images where no information existed about both interior and exterior orientation. An elegant and well-established technique of relating image and object is by the use of (1).

$$x + \Delta x = \frac{K}{N} \qquad y + \Delta y = \frac{P}{N}. \tag{1}$$

Where:

$$\Delta x = x^{-}(k_1 r^2 + k_2 r^4 + k_3 r^6) + p_1(r^2 + 2x^{-2}) + 2p_2 x^{-} y^{-}$$

$$\Delta y = y^{-}(k_1 r^2 + k_2 r^4 + k_3 r^6) + 2p_1 x^{-} y^{-} + 2p_2 (r^2 + 2y^{-})$$

$K = L_1X + L_2Y + L_3Z + L_4$

$N = L_9X + L_{10}Y + L_{11}Z + 1$

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$P = L_5X + L_6Y + L_7Z + L_8$

$\bar{x} = x - x_o$

$\bar{y} = y - yo$

$r^2 = (\bar{x} + \bar{y})^2$

Equation (1) results from the equation of the central perspective in a trivial manner; Δx, Δy are systematic deformations of the image, i-e. deviations from the central perspective. Equation (1) is rewritten to serve in a least squares formulation relating known control points to image coordinates measurements as shown in (2).

$$v_x = c_1 L_1 + c_2 L_2 + c_3 L_3 + c_4 L_4 + c_9 L_9 + c_{10} L_{10} + c_{11} L_{11} + c_{12} - \Delta x$$

$$v_y = c_5 L_5 + c_6 L_6 + c_7 L_7 + c_8 L_8 + c_{14} L_9 + c_{15} L_{10} + c_{16} L_{11} + c_{17} - \Delta y \quad (2)$$

Where

$c_1 = X/A$	$c_7 = Z/A$	$c_{13} = y.(X/A)$
$c_2 = Y/A$	$c_8 = 1/A$	$c_{14} = y.(Y/A)$
$c_3 = Z/A$	$c_9 = x(.X/A)$	$c_{15} = y(Z/A)$
$c_4 = 1/A$	$c_{10} = x.(Y/A)$	$c_{16} = y/A$
$c_5 = X/A$	$c_{11} = x(Z/A)$	
$c_6 = Y/A$	$c_{12} = x/A$	

and $A = L_9X + L_{10}Y + L_{11}Z + 1$

III. MIXED AND PARAMETRIC LEAST- SQUARES ADJUSTMENT TECHNIQUES

There are different techniques to be applied, when adjusting the geodetic and photogrammetric networks by the least–squares principle. Each one depends on a type of the mathematical models. The results of the adjustment are: the adjusted observations, the vector of residuals, the vector of the adjusted unknowns (parameters), and the corresponding covariance matrices. In this study, the mixed least–squares adjustment will be discussed. Then, the parametric least–squares adjustment technique will be given, as a special case of the mixed adjustment technique.

A. The Mixed Least–Squares Adjustment

The method of least square adjustment tries to solve for an optimal estimate of both the coordinates and residuals by minimizing the sum of squares of the weighted residuals. Here, the mathematical model has the form given in (3).

$$F_{c,1}(X_{u,1}, L_{n,1}) = 0.00 \quad (3)$$

Where

F: denotes the functional relationships between X and L

C: number of all possible independent functions relating the vectors X, L in the model.

U, n: number of unknowns and the observations respectively.

This model is known as mixed mathematical model, combined mathematical model, or condition equations with parameters.

Assume X_O is the vector of the approximate values of parameters, then:

$$X = \bar{X} - X_O \quad (4)$$

Where

X = The corrections to be added to the approximate parameters.

$\bar{X}$ = The vector of the adjusted parameters.

If L is the vector of the observations, the residual will be given by (5):

$$V = \bar{L} - L \quad (5)$$

Where

$\bar{L}$ The adjusted values of the observed quantities.

Then, the mathematical model is written as shown in (6).

$$F(X + X_o, L + V) = 0.0 \quad (6)$$

After linearization, the model will be as (7):

$$B_{c,n} V_{n,1} + A_{c,u} X_{u,1} + W_{c,1} = 0.0 \quad (7)$$

Where

$$B_{c,n} = \left.\frac{\partial F}{\partial L}\right|_{X_o, L}$$

$$A_{c,u} = \left.\frac{\partial F}{\partial X}\right|_{X_o, L}$$

$$W_{c,1} = F(X_o, L)$$

The coefficient matrices B, A and the misclosure vector W, are evaluated at the approximate parameters and the observed values.

A-1 The Minimization of the Mathematical Model:

To apply the least–squares principle (VTPV = min.), to the linearized model (BV +AX + W = 0.00), the unknown vector Kc,1, which is called the Lagrange multipliers is introduced. Then, the quadratic form ϕ = VTPV, can be written as (8):

$$\phi(V,K,X) = V^T PV - 2K^T(BV + AX + W) \quad (8)$$

To find the minimum of the variation function, the derivatives of ϕ with respect to V, K and X are set to zeros, as introduced in (9),(10),(11):

$$\frac{1}{2}.\left(\frac{\partial\phi}{\partial V}\right)^T = PV - B^T K = 0.00 \quad (9)$$

$$\frac{1}{2}.\left(\frac{\partial\phi}{\partial K}\right)^T = BV + AX + W = 0.00 \quad (10)$$

$$\frac{1}{2}.\left(\frac{\partial\phi}{\partial X}\right)^T = -A^T K = 0.00 \quad (11)$$

From (9), the residuals will be as (12):

$$V = P^{-1} B^T K \quad (12)$$

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After some substitutions and manipulations using (10), (11) and (12), it can be found that:

$K = - M^{-1} (A X + W)$ (13)

With

$M = B P^{-1} B^T$ (14)

Finally

$(A^T M^{-1} A) X = - A^T M^{-1} W$ (15)

Or: $Nm\ X = - Um$

Where : $N_m = (A^T M^{-1} A)$ & $U_m = A^T M^{-1} W$

Note that :

N_m = is referred as the normal equation matrix related to the mixed adjustment.

U_m = is the constant term related to the mixed adjustment.

The estimated parameters are given by (16):

$$X = -N_m^{-1} U_m \quad (16)$$

A-2 Determination of the Cofactor Matrices

Since the vector, W is a random vector, and it is a function of the observations (L), by applying the covariance law, (17) can be written as:

$$Q_w = B P^{-1} B^T \quad (17)$$

Equation (18) can be written based upon (16) and (17):

$$Q_X = (A^T M^{-1} A)^{-1} = N_m^{-1} \quad (18)$$

Now, the vector of residuals V, can be written as (19):

$$V = [P^{-1} B^T M^{-1} A (A^T M^{-1} A)^{-1} A^T M^{-1} - P^{-1} B^T M^{-1}] W \quad (19)$$

Applying the covariance law on (19), the cofactor matrix of the residuals will be as (20):

$$Q_V = P^{-1} B^T M^{-1} [M - A (A^T M^{-1} A)^{-1} A^T] M^{-1} B P^{-1} \quad (20)$$

The adjustment observations will be as in (21):

$$\bar{L} = L + V = L + [P^{-1} B^T M^{-1} A (A^T M^{-1} A)^{-1} A^T M^{-1} - P^{-1} B^T M^{-1}] W \quad (21)$$

Finally, the cofactor of the observations will be as in (22)

$$Q_L = P^{-1} - Q_V \quad (22)$$

A-3 The A Posterior Variance- Covariance Matrix

Associated with the above estimates are their corresponding accuracy estimates. At first, the a posterior variance factor can be estimated by (23):

$$\hat{\sigma}_o^2 = \frac{\hat{V}^T P \hat{V}}{r} \quad (23)$$

Where

r: is the total redundancy of the system = c - u.

The a posterior variance-covariance $\hat{\Sigma}_{\hat{X}}, \hat{\Sigma}_{\hat{V}}\ and \hat{\Sigma}_{\hat{L}}$ matrices will be as given in (24), (25) and (26)

$$\hat{\Sigma}_X = \hat{\sigma}_o^2 Q_X \quad (24)$$

$$\hat{\Sigma}_V = \hat{\sigma}_o^2 Q_V \quad (25)$$

$$\hat{\Sigma}_L = \hat{\sigma}_o^2 Q_L \quad (26)$$

In the linearization of the mathematical model, the higher order terms are neglected. So, the resulting mathematical model is valid only for very accurate approximate parameters. To overcome the non–linearity effect, the solution must be repeated by updating the vector X_O of the approximate values of the unknowns. Such a process is known as iterative solution. The iteration will be continued until the values of the solution vector (the corrections), are equal or less than a pre–specified precision limit.

B. The Observation Equation Least-Square Adjustment

In this model, the vector of the adjusted observations L is related as a function of the unknown parameter vector X (usually the coordinates) by (27):

$$\bar{L}_{n,1} = F_{n,1} (\bar{X}_{u,1}) \quad (27)$$

Where

$$\bar{L} = L + V = F(X_O + X)$$

$$\bar{X} = X_O + X$$

After linearization, this model will be in the form of (28)

$$V_{n,1} = A_{n,u} X_{u,1} + W_{n,1} \quad (28)$$

Here, the elements of B will be equal (- I). Substituting by B = - I in the results of the mixed adjustment, the results of the parameters adjustment will be in the form of (29)

$$X = - (ATPA)^{-1} (A^T PW) = - N U \quad (29)$$

Where

$N = (A^T P A)^{-1}$

$U = A^T P W$

$$\hat{\sigma}_o^2 = \frac{V^T PV}{n-u} \quad (30)$$

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$$\hat{\Sigma}_X = \hat{\sigma}_o^2 (A^T P A)^{-1} \quad (31)$$

Also, the covariance matrices of the adjusted observations and the estimated residuals will be as given in (32) and (33)

$$\Sigma_{\bar{L}} = A \Sigma_X A^T \quad (32)$$

$$\Sigma_V = \hat{\sigma}_o^2 (P^{-1} - A N^{-1} A^T) \quad (33)$$

There are two cases related to the normal equation matrix N:

a. the constrained adjustment: where N is of full rank (non–singular, i.e. det (A ≠ 0), then it will have the regular inverse $\mathbf{N^{-1}}$.

b. the rank deficient matrix N (singular, i.e. det (A = 0) and the solution is obtained by the generalized inverse theories.

IV. RESEARCH METHOD

To evaluate the two different least–squares adjustment techniques, numerical comparison will be made between both mixed and parametric adjustment techniques as applied to solve the system of direct linear transformation equations. In this study, simulation test field has been applied to verify the numerical comparison between the two different least – squares adjustment techniques. A total of 42 object points, forming cube with side of 8 meters, will be used as control and check points. Eighteen of such points will be taken as ground control points, while the remaining will be considered as check points. The frontal side of this cube will be photographed from ten different camera stations. The photo coordinates of each camera position are computed using the space coordinates of these object points, the suggested interior and exterior orientations for each photograph. The computed photo coordinates are given random error of N (0,σ) to simulate the field measured photo coordinates, where σ is the precision in measuring the photo coordinates in micron. The photo coordinates are assumed to be measured with precision of 3μm. The space coordinates of the object points are recomputed using both mixed and parametric methods using the simulated photo coordinates. The above computations will be processed several times, for both mixed and parametric adjustment techniques. In this research, ten different samples of generated random errors are prepared to be used for both adjusted techniques. This means that both adjustment will be recomputed eighteen times for each selected number of ground control points. It is to be mentioned that, in the double station geometry, the solutions will be repeated, using different number of ground control points, starting with eight and ending up with eighteen by increasing one point each time. It is also to be noted that, in the multi station geometry, the solutions will be repeated using only fourteen of ground control points, starting with two camera stations and ending up with ten camera stations by increasing one camera station each time. Using the above described method, one gets 36 values for the object space coordinates (Xphi, Yphi, Zphi) for each twenty four check points, as estimated from the two different least- squares adjustment techniques for each specific number of ground control points. The RMS of the object point coordinates using the simulation techniques are computed from the difference between the true coordinates of the check points and the recomputed coordinates of these check points, and the average values for all check points are determined. From the above strategy, one can to have 36 values for the three-dimensional object coordinates (X, Y, Z) for each one of the used twenty-four check points, as computed from both least-squares adjustment techniques for each used number of ground control points. The RMS will be used for the accuracy assessment of both mixed and parametric adjustment techniques as applied to the direct linear transformation equations.

V. ASSESEMENT OF ACCURACY

According to Hottier 1976, there are two methods can be used to evaluate accuracy: one can evaluate accuracy by using check measurements and determining from these check measurements the value of appropriate accuracy criteria; and one can use accuracy predictors. In this study, check measurements will be used to evaluate accuracy.

VI. RESULTS DISCUSSIONS

Both mixed and parametric least square adjustment techniques have been applied. The maximum residual in the X, Y and Z-direction, and the maximum spatial residual among the n check points were calculated. All of the obtained results of the RMX, RMY, RMZ and RMXYZ are presented. A MATLAB program was prepared for computation of the spatial coordinates (X, Y, Z) of the n check points, the maximum residual (RMX, RMY, RMZ) and the maximum spatial residual (RMXYZ) among the check points and the variance-covariance matrix of the parameters. RMX, RMY, RMZ and RMXYZ for the object coordinates of the check points are presented graphically in figures 1 to 5. The evaluated standard deviation SD for the object coordinates is also be presented in tabular form as shown in table 1. It is to be mentioned that, the standard deviation (SD), as calculated from the least-squares adjustment will be considered as a measure of precision and the obtained results of the RMX, RMY, RMZ and RMXYZ will be considered as a measure of accuracy. Two curves appear on each diagram: one represents the mixed adjustment technique and the second represents the parametric adjustment technique.

Table (1) The Precision in Case of Two Station Geometry

Pts	stdmX (mm)		stdmY (mm)		StdmZ (mm)	
	Parametric	Mixed	Parametric	Mixed	Parametric	Mixed
11	2.916534	2.915381	2.707631	2.711365	2.343534	2.349850
12	3.479580	3.450878	1.414740	1.403892	2.334894	2.317875
13	1.748452	1.745710	0.825382	0.824306	1.066870	1.065808
14	1.492035	1.490482	0.763721	0.763177	0.931810	0.931172
15	1.456195	1.455107	0.641480	0.641087	0.932397	0.932251
16	1.195809	1.193171	0.633456	0.632392	0.677932	0.676630
17	1.202918	1.199134	0.574819	0.573515	0.603193	0.601787
18	1.303003	1.299402	0.606082	0.604754	0.612290	0.610746

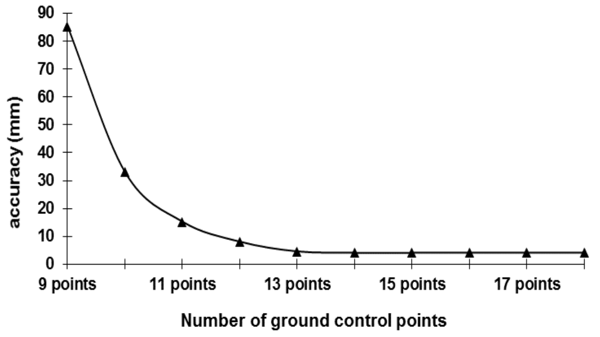


Figure (1): The relation between the accuracy and the number of ground control points obtained from the mixed adjustment technique

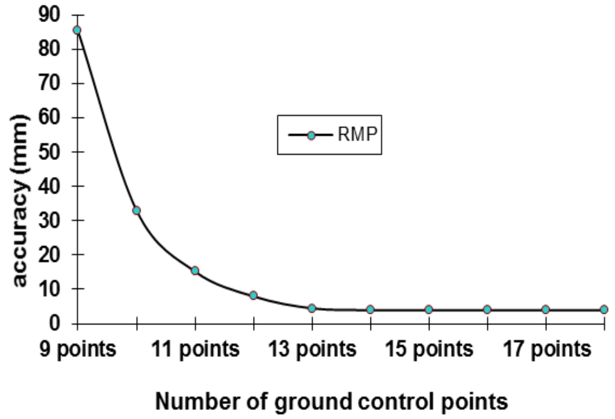


Figure (2): The relation between the accuracy and the number of ground control points obtained from the parametric adjustment technique

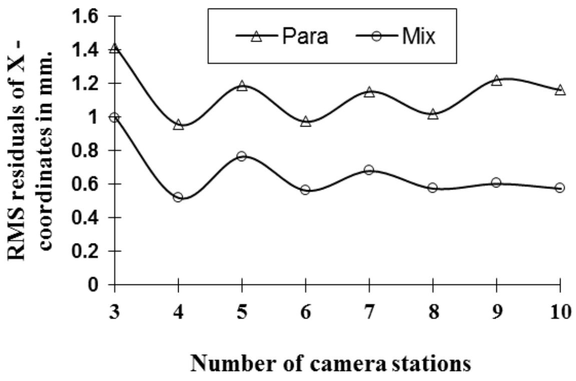
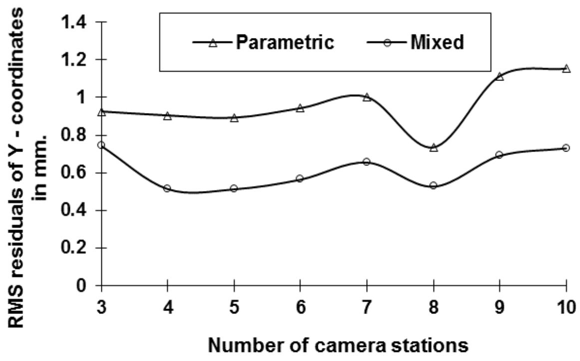
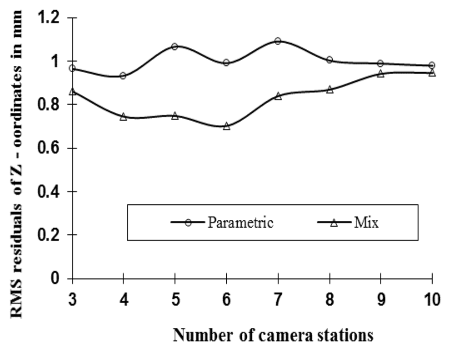


Figure (3): The relation between RMS residual of X-coordinates and the number of ground control points



Figure(4): The relation between RMS residual of Y-coordinates the number of Camera Stations



Figure(5): The relation between the RMS residual of Z-coordinates the number of camera

Examining of the presented results in table (1) and in figures (1) to (5) yields the following remarks:

1. In both mixed and parametric least squares adjustment techniques, the estimated values of the standard deviation SD for the object coordinates are almost coincide with its respective values of the RMX, RMY, RMZ and RMXYZ which shows no biases of the used mathematical model.
2. In both mixed and parametric least squares adjustment techniques, the accuracy decreases with the increase of the number of ground control points until the number of ground control points reaches to limit number (14), after that there is any significant improvement with the increasing of the ground control points.

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3. The required computing time for solving the used photogrammetric model using both adjustment techniques is relatively close to each other.
4. In the double station geometry, both techniques give almost the same accuracy.
5. In the multi station geometry, the mixed adjustment technique gives significant improvements in accuracy against the corresponding values obtained from the parametric adjustment technique as the following:

- In X-direction: The improvement in accuracy is varied between 30 % to 50%
- In Y-direction: The improvement in accuracy is varied between 20 % to 43%
- In Z-direction: The improvement in accuracy is varied between 22 % to 37%

VII. CONCLUSIONS

From the obtained results, we can conclude that:

1. In both mixed and parametric least-squares adjustment techniques, there is a directly proportional between the number of ground control points and the estimated accuracy of the object coordinates until the number of ground control points reaches 14 point, after that, there is no any significant improvements when the number of ground control points is increased, and this result conform with the results of many others authors.
2. In both mixed and parametric least squares adjustment techniques, there is no bias of the used mathematical model.
3. In the double station geometry, the results obtained from the parametric least squares adjustment are relatively close to the corresponding results obtained from the mixed parametric least squares adjustment.
4. In the multi station geometry, the best estimated accuracy for the object space coordinates is achieved when the mixed least squares adjustment technique was used for solving the direct linear transformation equations in comparison with the estimated accuracy from the parametric least squares adjustment technique.

From all of the above, it is clear that the mixed adjustment technique is preferred to the parametric adjustment technique, therefore it is recommended to adopt the mixed adjustment technique rather than using the parametric adjustment technique for solving the direct linear transformation equations.

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