International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-8 Issue-3S, February 2019

Enhanced Newton-Raphson Algorithm in Estimating Internal Rate of Return (IRR)

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ABSTRACT--- Internal Rate of Return (IRR) is the most compelling performance metric among measures for profitability of investments. The most efficient way to estimate it is by using iterative methods, four of the most popular of which are false position, bisection, secant and Newton-Raphson algorithms. Although Newton-Raphson method is the quickest among them, it does not, however, converge to the root if the user's guess initial input value is far from the true value of IRR. This study proposes an enhancement that gets rid of such user's guess input and makes input automatically generated, improves accuracy, lessens the number of iterations and shortens the runtime. The study finds that said enhancement, indeed, does not require user guess input and allows the algorithm to converge to the root with higher accuracy, fewer iterations and shorter runtime.

Keywords: IRR, Root-finding algorithm, Newton-Raphson algorithm, Convergence, Divergence

INTRODUCTION

Internal Rate of Return (IRR), the most compelling performance metric among financial indicators, is the discount rate that makes the net present value of a series of cash flows equal to zero, and is used heavily as a measure for the profitability of investments (Costello & Pecher, 2017) (Eric B. Storey, 2016) .

But its value cannot be determined analytically but can be estimated by using a complex root-finding algorithm (Sangah, Shaikh, & Shah, 2016) (Nijmeijer, 2015). Four of the most popular root-finding algorithms are Bisection, False position, Secant and Newton-Raphson (Smyth, 2015)(Ahmad, 2015)(Sultana & Shinday, 2016)(Salimi, Lotfi, Sharifi, & Siegmund, 2017) (Mancusi & Zoia, 2018)(Ruslan & Jaffar, 2017).

PROBLEM STATEMENT

There are other root-finding algorithms, but they have unacceptable drawbacks, such as user's guess initial value, non-convergence, computational cost, lesser accuracy and lesser speed. Although the Newton–Raphson algorithm is one of the most widely-used algorithms for finding roots due to its very quick quadratic convergence (Salimi et al., 2017)(Liang, Shi, & Chung, 2017), it has, however, big drawbacks, particularly uncertain convergence and division by zero (Yamamoto, 2000)(Gholami & Aghamiry, 2017)(Mujahed & Elshareif, 2017).

RATIONALE OF THE STUDY

The proponents are motivated to solve the problems of Newton-Raphson algorithm, such as its inability to converge to a true root caused by the user's guess initial input being far from the true root.

THE AIM OF RESEARCH

The general objective of this study is to enhance the Newton-Raphson algorithm to specifically estimate IRR. To achieve the general objective, the following are the specific objectives of this study: 1) To modify said algorithm by injecting "IRR1 ← (**Σ**Ci)/Co)^(1/((((N -1)/2)+1))-1" to automatically produce an initial value close to a true root, thereby eliminating a guess from the user; 2) To increase the accuracy of the Newton-Raphson algorithm in determining the IRR by reducing the value of the non-linear function NPV(IRR) to less than or equal to (<=) 0.0000000000001 (or 10E-13); and 3) To improve the speed of the algorithm by reducing the number of iterations of the original Newton-Raphson algorithm.

LITERATURE REVIEW

(Shestopaloff & Shestopaloff, 2013) claimed to have discovered that the largest root of the IRR equation, which necessarily produces the largest rate of return, is the most adequate solution of the IRR equation. Solving this long-standing problem, which is of very high practical and theoretical importance in finance, opens lots of new opportunities for developing new robust financial instruments and advanced analytical methods (Shestopaloff & Shestopaloff, 2013). Their study, however, in the first place did not show how the the largest root can be determined as doing so requires that all possible roots must be determined first. Moreover, they did not put forward their argument why the largest root is the right root. (Silalahi, Laila, & Sitanggang, 2017) discussed methods for finding solutions of nonlinear equations: the Newton method, the Halley method and the combination of the Newton method, the Newton inverse method and the Halley method. Computational results in terms of the accuracy, the number of iterations and the running time for solving some given problems are presented.

Manuscript published on 28 February 2019.
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However, though this algorithm is a combination of three (3) other algorithms, the problem of requiring an initial-value guess from the user still exists and can still cause divergence. (Zafar, Yasmin, Akram, & Junjua, 2015) constructed a new general class of derivative free n-point iterative methods of optimal order of convergence 2^(n−1) using rational interpolant. However, the new method still has the problem of requiring initial guess value from the user. Applying various classical root-finding algorithms to digital maximum power point tracking (DMPPT), (Chun & Kwasinski, 2011) proposed modified regula falsi method (MRFM) and applied it to photovoltaic (PV) applications, and claimed it as being faster than certain other methods and that convergence is ensured. However, the Modified Regula Falsi Method (MRFM) still requires two initial guesses which can cause non-convergence. The exponential interpolation and corrected secant formulas described in the paper of (Moten & Thron, 2013) are claimed to obtain more accurate results with less effort than the secant method, and can be used for hand calculation of IRR or for programming if needed. The corrected secant method also provides an estimate of the uncertainty. However, the corrected secant method is complex. (El-Tahir & El-Otaibi, 2014) devised what they believed as accurate mathematical formula that calculates precisely the IRR. However, the alternative formula presupposes only one value of the initial project return and only one for cost, which is not the case in real situations as there exist more than one value for each year in the life span of a project. (Qiao & Zhang, 2010) used Particle Swarm Optimization (PSO) and Ant Colony Optimization (ACO) algorithms to estimate IRR. However, these algorithms' speeds and accuracy levels are not good enough. (Ruslan & Jaffar, 2017) determined IRR for a diminishing musyarakah model by applying bisection and secant methods. However, bisection and secant algorithms are slow in converging, if they converge at all. Criticizing Newton-Raphson algorithm for its divergence and division-by-zero drawbacks, (Mujahed & Elshareif, 2017) developed what they call a simplified IRR approach treating cash inflows as constant and positive. Though the formula appears simpler, it is, however, actually still complex, so much so that it still cannot be solved analytically or by a closed-form expression and still needs a root finding algorithm. Moreover, the said simplified formula presupposes that cash inflows are all positive and identical, which is disadvantageous, considering that such situation is not the case in the real world. Believing that using trial and error method to calculate IRR of a complex investment portfolio in a sequential method can take several hours, (Casturi, 2014) compared experimentally parallel computations on GPUs, regular sequential method and a database-driven framework. While SQL approach is found to be the fastest, it, however, sacrificed simplicity and is computationally costly. (Gisin & Volkova, 2017) find IRR as an existing IT problem so much so that they have developed a fuzzy approach to solving it. They, however, have not presented the speed and accuracy of their approach. (Rangel et al., 2016) consider the calculation of IRR so complex that they recommend using a simplified closed-form approach. This, however, has low accuracy.

METHOD OF RESEARCH & RESULTS

This study proposes the Enhanced Newton-Raphson Algorithm, as shown in Figure 1.

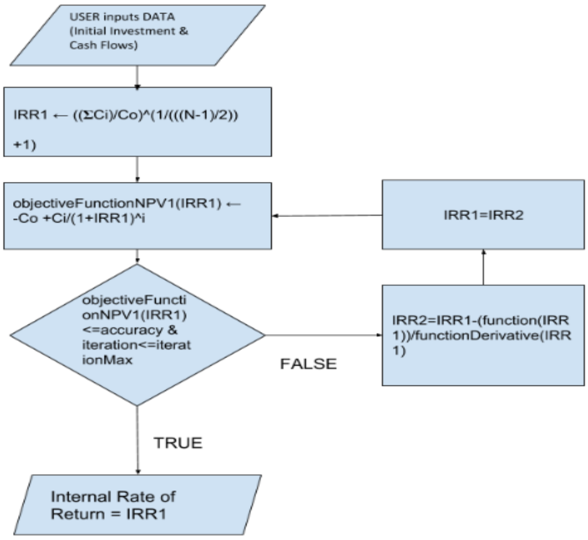

Figure 1. Design Flow of the Proposed Study

The enhancement consists of 1) the removal of user's guess initial value of the IRR, 2) making the initial value automatic by deriving the initial value from the input of cash flows, Co, C1, …, Cn, and 3) using the formula IRR1 ← (((∑Ci)/Co)^(1/(((N-1)/2) + 1)) – 1), as shown in Figure 1 above.

ANALYSIS AND DISCUSSION

Result of Automatic Initial Value Computation:

As demonstrated by Table 1 below, the experiment shows that the enhanced Newton-Raphson algorithm automatically computes initial value, instead of waiting for user's guess initial input value in order for the Newton-Raphson algorithm to operate. What the enhanced algorithm simply needs are the data consisting of the principal amount, also known as initial investment or outlay, C0, and the subsequent cash flows, C1, C2, …, Cn. These data serve as the respective coefficients of the terms of the function to be differentiated by using Differential Calculus.

In this study, using the data in the study of (Qiao & Zhang, 2010) and (Patrick & French, 2016), the value of IRR was estimated without any need for an initial guess value from the user:

(Qiao & Zhang, 2010) used the following problem data: Each year, the beginning investment of a hydraulic engineering project, in its four-year construction period, is 70×106 yuan; the usage period of the engineering project is 34 years; the annual benefit of the engineering is 45×10^6 yuan and the annual operation cost is 10×10^6 yuan. Then the IRR needs to be estimated for engineering economic analysis. The end of the fourth year is the discount base year. In other words, C0, C1, C2 and C3, respectively, equal -70 Million Yuan; C4 equals 0; each of C5, C6, …, and C38 equals 35 Million Yuan.

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The IRR is estimated as 0.0946180708797336, using the enhanced version.

(Patrick & French, 2016) uses the following data: Initial investment, C0 = 145; C1 = C2 = C3 = C4 = 100; and C5 = -275.

Result on Accuracy:

The new algorithm can estimate the value of IRR, with a very high degree of accuracy, given the data C0, C1, C2, …, Cn. Given the same data in (Qiao & Zhang, 2010) study, function of IRR, f(IRR), equals -3.5527136788005 X 10^(-13); that is, it has 14 decimal places, or is practically zero (0). Parenthetically, a function of IRR, f(IRR), equal to zero (0), is a condition where the determined value of IRR is the exact value of IRR. So, in the Qiao-Zhang data, the IRR of 0.0946180708797336 (or 9.46180708797336%) is almost exact as f(IRR) is approximately zero.

Result on Speed:

Given the same data in (Qiao & Zhang, 2010) study, the enhanced algorithm's speed, as shown in the Table 1 below, is observed at only 6 iterations at merely 200 milliseconds, without user's initial guess, compared to the original algorithm's infinite IRR, inability to converge in at least 1000 iterations, with user's initial guess of 1.

Table 1. Enhanced Newton-Raphson vs. Old Newton-Raphson

ENHANCED NEWTON-RAPHSON VS. OLD NEWTON-RAPHSON							
ORIGINAL / ENHANCED	PROBLEM	INITIAL GUESS	IRR	ITERATIONS	RUNTIME (in ms)	ACCURACY: f(IRR)	REMARKS
OLD	Qiao-Zhang	-1	0 (Error)	NA	NA	NaN	Error returned
		1	Infinity	No convergence in 1000	NA	NA	No convergence
		0.25	0.094618070879726200	29	865	2.09E-11	Too many iterations
		0.3	-1.999999999999340000	26	1827	4.82E-10	Unrealistic IRR value
	Patrick-French	-1	0 (Error)	NA	NA	NaN	Error returned
ENHANCED	Qiao-Zhang	Guess Not Needed	0.0946180708797336000	6	200	-3.5527136788005E-13	Quick convergence
	Patrick-French	Guess Not Needed	0.0878282738497603000	6	101	-8.8391516328556400000E-12	Quick convergence

CONCLUSION

The proposed Enhanced Newton-Raphson algorithm, modifying the original algorithm by injecting "IRR1 ← (ΣCi)/Co)^(1/((((N -1)/2)+1))-1" and allowing data input in the form of cash flows, can automatically produce an initial value close to a true root, thereby eliminating a guess initial value from the user.

The accuracy of the Enhanced Newton-Raphson algorithm in determining the IRR is improved by reducing the value of the non-linear function NPV(IRR) from greater than or equal to (>=) 0.00000001 (or 10E-8) to less than or equal to (<=) 0.0000000001 (or 10E-10).

The speed of the Enhanced Newton-Raphson algorithm is improved by reducing the number of iterations and runtime of the original Newton-Raphson algorithm.

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