



# Free Vibration Response of Four-Parameter Functionally Graded Thick Spherical Shell Formulation on Higher-Order Shear Deformation Theory

Raparthi Srilakshmi, Ch. Ratnam, Chandra Mouli Badiganti

**Abstract:** This paper emphasizes on the free vibration (FV) responses of functionally graded thick spherical shell in rectangular form using traditional mathematical formulation on finite element method and governed by Higher order shear deformation theory (HOSDT). A functionally graded spherical shell made up of metal-rich on the top surface and in contrast, base surface of the model is ceramic-rich. The FG volume fraction of four-parameter power-law material constituents assumed in the thickness direction. To highlight the potential for the current method, convergence studies, and validation tests performed to establish the stability and accuracy attained by the current approach. The parametric studies presented to scrutinize the influence of choice of four parameters employed through power-law distribution. The eminence effect of spherical shell geometrical properties, and different types of support conditions, skew angle on the FV behavior of non-dimensional frequency responses examined in detail.

**Keyword:** Free vibration, HSDT, Finite element method, Spherical shell.

## I. INTRODUCTION

The continuous evaluation of develop materials and to optimize the structural design have received significant attention in many engineering areas and manufacturing industries. FGMs are a structurally advanced new class of composites. The concept originated in the 1980s in Japan. FGMs continuous gradation behavior of material composition (metal and ceramic) and microstructure. FGMs preferred in many sectors because of their mechanical and heat-shielding characteristics while maintaining structural integrity and reducing stress concentrations. Many research works contributed to bending and buckling analysis. The vibration behavior of shells plays a crucial role in design aerospace equipment, spacecraft, rockets, missiles, containers, hydraulic structures, submarines, ships, storage tanks.

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Loy et al. [1] examined the vibration of cylindrical with FGMs constituents with- general boundary conditions. The analysis considered based-on Love's Shell theory and Rayleigh-Ritz's method. A similar approach followed Pradhan et al. [2] studied FG Cylindrical Shells under different boundary conditions formulation on classical plate theory. Reddy [3] presented an analysis of Functionally graded plates. Ng et al. [4] introduced the dynamic stability analysis of FG shells under harmonic axial loading and Bolotin's approximation. K. M. Liew et al. [5] studied a three-dimensional vibration analysis of spherical shells panel subjected to various boundary conditions employing the P-Ritz method.

J.N. Reddy and Z.Q. Cheng [6] studied a spherical shallow shell in polygonal planform resting on a Winkler Pasternak elastic foundation. Hu et al. [7] studied the natural frequencies of rotating twisted and open conical shells. E. Artioli and Viola [8] presented the FV analysis of spherical caps using the Generalized Differential Quadrature (GDQ) procedure and the FEM approach. Arciniega and J.N. Reddy [9] described, Nonlinear analysis for functionally graded shells. The formulation is on the first-order shear deformation theory (FSDT) with seven independent parameters.

Zhao et al. [10] analyzed the FV of functionally graded shells using the element-free KP-Ritz method. F. Tornabene and E. Viola [11], [12] investigated the dynamic behavior of vibration analysis of hemispherical domes and spherical shell panels using FSDT. Later on, the same authors developed a 2-D solution for free vibrations of parabolic shells using a GDQ, and the FSDT is used to analyze the above moderately thick structural elements. Numerical solutions with the ones obtained using commercial programs such as Abaqus, Ansys, Femap/Nastran, Straus, Pro/Mechanica used. F. Tornabene [13] investigated based on the FSDT. Dynamic behavior of moderately thick FG conical, cylindrical shells, and annular plates with two different power-law distributions presented. Tornabene et al. [14]– [16] extended the GDQ procedure for the FV analysis of FG doubly-curved panels and shells of revolution with classical boundary conditions.

M.H.yas and B. Sobhani Aragh [17] studied three-dimensional analysis for the thermoelastic response of functionally graded fiber-reinforced cylindrical panel.



Amabili and J.N. reddy [18] studied higher-order deformation nonlinear theory for the large amplitude of forced vibration of laminated doubly curved shells. Mohammad Talha and B.N Singh [19] studied static response and FV analysis of FGM plates using higher order shear deformation theory and conjunction with finite element method.

Shen and Z.X. Wang [20] described based on HSDT, comparison studies of two kinds of micromechanics models, namely Voigt's and Mori-Tanaka models for vibration analysis of FG plates. Neves A.M.A et al. [21] presented with FV problems of FG spherical as well as cylindrical shell panels with all edges clamped or simply supported. The analysis is performed by radial basis functions collocation, according to a HSDT that accounts for through-the-thickness deformation. Discretization method-based on the FSST.

V. R. Kar and S. K. Panda [22]. investigated the vibration and thermal buckling characteristics of FG single/double curved panels under linear and nonlinear behaviors temperatures fields are studied. Kumar et al. [23] developed for FV analysis of laminated composite skew cylindrical shells. A  $C_0$  finite element formulation based on using HSDT.

C. Zhang et al. [24] presented the improved Fourier series method based on Hamilton's principle to investigate the vibration characteristics of circular cylindrical double-shell structures with different boundary conditions. Devesh Punera and Tarun Kant [25] developed FV of FG open cylindrical shells based on several refined higher-order displacement models. Mouli and Ramji Koonan [26]-[28] examined the influence of different parameters on the FV behavior of FG Skew shallow curved panels.

In the present work focused on the FV characteristics of FGM rectangular spherical shell. The formulation based on Higher-order deformation theory conjunction with the finite-element method. The comprehensive parametric studies carried out to examine the influence of power-law distribution and Choice of four parameters on material composition in terms of volume fraction constituents. and the significance of Geometric parameters is that the shell aspect ratio, thickness ratio, and curvature ratio. and the impact of different boundary conditions, skew angle, on the non-dimensional frequency responses, are studied.

## II. FGM MATERIAL PROPERTIES AND FOUR-PARAMETER POWER-LAW DISTRIBUTION PARAMETERS

In this part, the variation of volume fraction through the different values of power-law exponent and choice of distribution parameters with classic, with reference surface to the shell (Symmetric) and without reference surface (asymmetric) volume fraction profiles illustrated. The material properties of the FG spherical shell are assumed to vary throughout the thickness, where  $\vartheta_m$  and  $\vartheta_c$  are the volume fractions of metal and ceramic. Similarly, in which subscripts c and m represent the ceramic and metal constituents, as in (1a), (1b), and (1c) mechanical properties are Young's modulus  $E(\xi)$ , density( $\rho(\xi)$ ), and Poisson's ratio( $\nu(\xi)$ ) vary continuously to achieve smooth gradation of material phase through the spatial direction ( $\xi$ ) expressed in the form of a linear combination. The sum of the volume fraction of the constituent materials should be equal to one, as in (2).

$$E(\xi) = (E_c - E_m)\vartheta_c + \vartheta_m \quad (1a)$$

$$\rho(\xi) = (\rho_c - \rho_m)\vartheta_c + \rho_m \quad (1b)$$

$$\nu(\xi) = (\nu_c - \nu_m)\vartheta_c + \nu_m \quad (1c)$$

$$\vartheta_c + \vartheta_m = 1 \quad (2)$$

According to the following power-law function as in (3) Where  $\varphi$  is the power-law exponent varying as  $0 \leq \varphi \leq \infty$ , and the distribution parameters u, v, w and different values of power law exponent generates material variation profiles through the thickness direction in terms of volume fraction. For example, FG constituents of the shell thickness demonstrated through four parameter power law distribution. the material distribution in the FGM shell is continuously varied, such that the bottom surface (-h/2) of the structure is pure ceramic. In contrast, the top surface (+h/2) pure metal, by setting  $u = 1$  and  $v = 0, w=0$ , as in (3).

$$FGM_{(u/v/w/\varphi)}: \vartheta_c = \left( 1 - u \left( \frac{1}{2} + \frac{\xi}{h} \right) + v \left( \frac{1}{2} + \frac{\xi}{h} \right)^w \right)^\varphi \quad (3)$$

Fig.1 shows the classic volume fraction profiles, power-law distributions are considered for the volume fraction of the ceramic. The first distribution FGM ( $u=1, v=0/w=1/\varphi$ ), the material composition is continuously varied, such that the bottom surface of  $\xi/h=-0.5$  of the shell's ceramic-rich. In contrast, the top surface ( $\xi/h=0.5$ ) is metal-rich.

Fig.2 shows the significance of the various power-law distribution of patterns by modifying the parameters u, v, w, and  $\varphi$  for the given constituents of volume fraction. by varying the power law exponent ( $\varphi$ ) and symmetric respect to the reference surface ( $\xi/h=0$ ) of the shell. by setting the first four parameter power law distribution FGM ( $u=1, v=1$  and  $w=3$ ).

Fig.3 illustrate profiles are not symmetric with respect to the reference surface ( $\xi/h=0$ ) of the shell. asymmetric profiles obtained by setting FGM ( $u=1, v=1, w=5/\varphi$ ).

Figs.4-6. depicts varying the parameters u, v, w. These material profiles characterized by the fact that one of the shells surfaces the top or bottom surface presents a composed of two constituents. For example, by setting the values as in (3) and the power-law distribution is FGM ( $u=1, v=0.4, w=3$ ) and different quantity values of the power-law exponent. From the design point of view, it is essential to know the top surface is of the shell  $\xi/h=0.5$  is ceramic or metal. If the bottom surface  $\xi/h=-0.5$  is metal or ceramic- rich. One of these surfaces presents a mixture of two constituents. It is worth noting that types four-parameter power-law distributions as enunciated by F. Tornabene [13]-[16], and the author provided more detailed descriptions about the material variation profile of FGMs.

The primary purpose of this section The Voigt's rule is employed to estimate the ceramic volume fraction. The four-parameter power-law distribution affects the desired volume fraction of the material on the top and bottom surface of the structure.

Thus, FGM material properties of layers change continuously and smoothly in the thickness direction. The influence of the choice of four material parameters  $u$ ,  $v$ ,  $w$ ,  $\phi$ , and to study the variation of the volume fraction of the

material constituents and effects of various forms of classic, symmetric and asymmetric material pattern profiles which change the mechanical behavior of a structure.

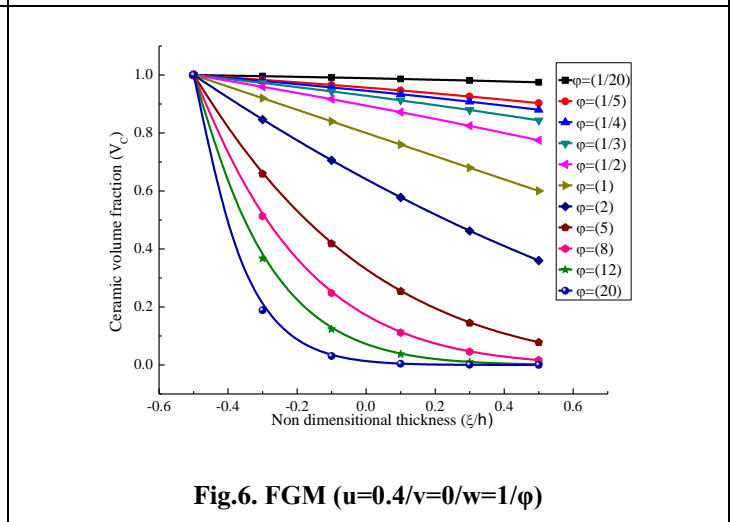
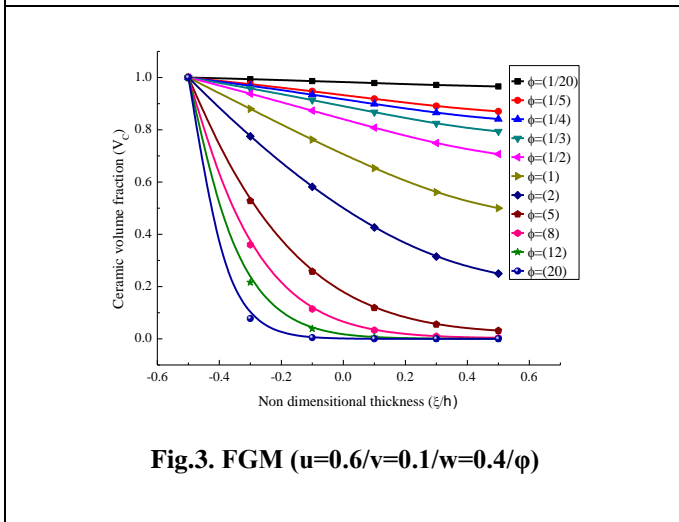
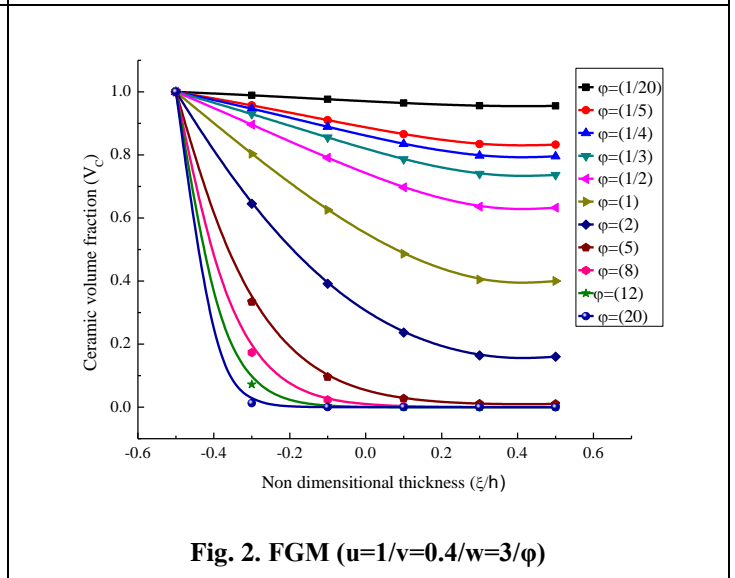
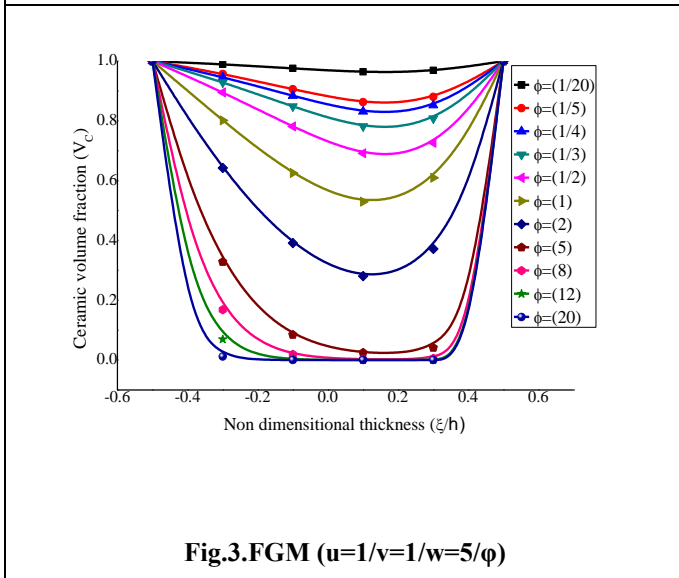
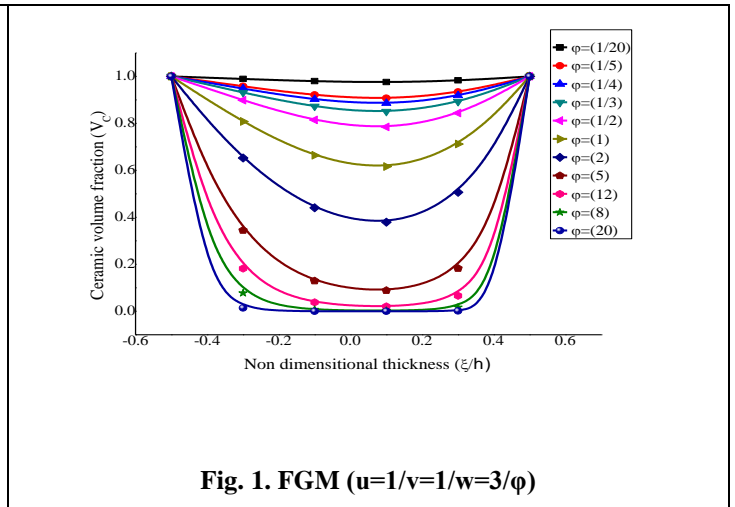
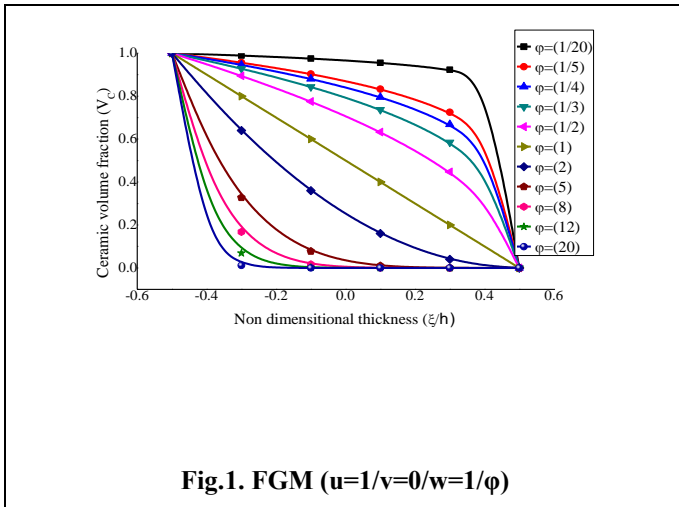


Fig. 1-6. Variation of ceramic volume fraction ( $V_c$ ) through the FG structure thickness ( $\xi$ ) for different values of three parameters  $u$ ,  $v$ ,  $w$ , and  $\phi$

### III. FRAMEWORK OF GOVERNING EQUATIONS

#### 3.1 Kinematic Paradigm and numerical procedure

In this framework, the formulation procedure for governing equations obtained by a general shallow curved spherical shell is represented by a, b, and thickness h, in x, y directions, as shown in Fig.7. Here, Rx and Ry are the radii of curvature along x and y direction, respectively. The TSDT mid-plane kinematics model utilized to define the global displacements ( $\chi, \lambda, \psi$ ) at any point in terms of mid-plane displacements ( $\chi_0, \lambda_0, \psi_0$ ),

rotations ( $\theta_x, \theta_y$ ) and higher-order ( $\chi_0^*, \lambda_0^*, \theta_x^*, \theta_y^*$ ) terms, as per Pandya & Kant, [30].

$$\left. \begin{aligned} \chi &= \chi_0 + \xi \theta_x + \xi^2 \chi_0^* + \xi^3 \theta_x^* \\ \lambda &= \lambda_0 + \xi \theta_y + \xi^2 \lambda_0^* + \xi^3 \theta_y^* \\ \psi &= \psi_0 \end{aligned} \right\} \quad (4)$$

This kinematic model, (5) can be rewritten in the matrix form as in (5)

$$\{\delta\} = [F]\{\delta_0\} \quad (5)$$

where,  $\{\delta\} = [\chi \ \lambda \ \psi]^T$  and  $\{\delta_0\} = [\chi_0 \ \lambda_0 \ \psi_0 \ \theta_x \ \theta_y \ \chi_0^* \ \lambda_0^* \ \theta_x^* \ \theta_y^*]^T$  are the global and mid-plane displacement vectors.  $[F]$  contains the thickness coordinate functions, as expressed here in (6)

$$[F] = \begin{bmatrix} 1 & 0 & 0 & \xi & 0 & \xi^2 & 0 & \xi^3 & 0 \\ 0 & 1 & 0 & 0 & \xi & 0 & \xi^2 & 0 & \xi^3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

The shallow curved shell is shown in Fig.7(ii) with sides a and b. To constraint the oblique edges, the local displacement vector is required to transform to global via transformation matrix  $[F]$  and expressed in cosine ( $l$ ) and sine ( $m$ ) terms. The displacement transformation revealed as

$$\begin{pmatrix} \chi_0 \\ \lambda_0 \\ \psi_0 \\ \theta_x \\ \theta_y \\ \chi_0^* \\ \lambda_0^* \\ \theta_x^* \\ \theta_y^* \end{pmatrix} = \begin{bmatrix} l & -m & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ m & l & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & l & -m & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & l & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & l & -m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m & l & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & l & -m \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m & l \end{bmatrix} \begin{pmatrix} \chi_0' \\ \lambda_0' \\ \psi_0' \\ \theta_x' \\ \theta_y' \\ \chi_0'^* \\ \lambda_0'^* \\ \theta_x'^* \\ \theta_y'^* \end{pmatrix} \quad (7)$$

(7) can also be written as

$$\{\Omega_0\} = [F]\{\Omega_0'\} \quad (8)$$

where,  $\{\Omega_0'\}$  is the displacement field defined in the local coordinates.

The strain-displacement equation for any general shallow thick plate can be written as (Kar and Panda, [22])

$$\left\{ \begin{aligned} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{aligned} \right\} = \left\{ \begin{aligned} \frac{\partial \chi}{\partial x} + \frac{\psi}{R_x} \\ \frac{\partial \lambda}{\partial y} + \frac{\psi}{R_y} \\ \frac{\partial \chi}{\partial y} + \frac{\partial \lambda}{\partial x} \\ \frac{\partial \chi}{\partial z} + \frac{\partial \psi}{\partial x} - \frac{\chi}{R_x} \\ \frac{\partial \lambda}{\partial z} + \frac{\partial \psi}{\partial y} - \frac{\lambda}{R_y} \end{aligned} \right\} \quad (9)$$

By imposing the displacement terms (4) in the strain-displacement as in (9), the global strain tensor can be modified as

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{pmatrix} = \begin{pmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \\ \varepsilon_{xz}^0 \\ \varepsilon_{yz}^0 \end{pmatrix} + \xi \begin{pmatrix} k_x^1 \\ k_y^1 \\ k_{xy}^1 \\ k_{xz}^1 \\ k_{yz}^1 \end{pmatrix} + \xi^2 \begin{pmatrix} k_x^2 \\ k_y^2 \\ k_{xy}^2 \\ k_{xz}^2 \\ k_{yz}^2 \end{pmatrix} + \xi^3 \begin{pmatrix} k_x^3 \\ k_y^3 \\ k_{xy}^3 \\ k_{xz}^3 \\ k_{yz}^3 \end{pmatrix} \quad (10)$$

$$\varepsilon = \varepsilon^0 + \xi k^1 + \xi^2 k^2 + \xi^3 k^3 \quad (11)$$

where,  $\varepsilon^0, k^1, k^2$  and  $k^3$  are the mid-plane strain, curvature and higher-order terms respectively as per Kar and Panda [22].

(11) can be again rearranged as

$$\{\varepsilon\} = [F]\{\bar{\varepsilon}\} \quad (12)$$

where,  $\{\bar{\varepsilon}\} = [\varepsilon^0 \ k^1 \ k^2 \ k^3]^T$  is the mid-plane strain, and  $F = [I \ \xi I \ \xi^2 I \ \xi^3 I]$  is the thickness-coordinate matrix, in which  $I$  is the unit matrix of size  $5 \times 5$ .

### 3.2 Finite Element Approximations

In this section, the present FG shallow spherical panel is discretized based on a finite element modeling using a nine-node isoperimetric element with eighty-one degrees-of-the displacements defined in the mid-plane can in a nodal form as

$$\{\Omega_0\} = \sum_{i=1}^9 N_i \{\Omega_{0i}\} \quad (13)$$

where,  $\{\Omega_{0i}\}$  and  $N_i$  are the nodal displacement vector and the approximation function at  $i$ th node (Cook et al. [29]). Now, the total and geometric mid-plane strain vectors can be expressed as in (13) as

$$\{\bar{\varepsilon}\} = [B]\{\Omega_{0i}\} \text{ and } \{\bar{\varepsilon}_G\} = [B_G]\{\Omega_{0i}\} \quad (14)$$

where,  $[B]$  and  $[B_G]$  are the differential operators of the total and geometrical mid-plane strains respectively. By imposing (12)-(15) in the above governing equation, the equilibrium equation of the vibrated FG shallow shell panel is achieved and expressed in global form, as

$$([K] - [K_G] - \omega^2 [M])\Delta = 0 \quad (15)$$

where,  $[M] = [N]^T [m] [N]$  is the global mass matrix,

$[K] = [B]^T [D] [B]$  is the global stiffness matrix,

$[K_G] = [B_G]^T [D_G] [B_G]$  is the geometric stiffness

matrix and  $\omega$  and  $\Delta$  are the natural frequency and eigenvalue type the corresponding eigen vectors.

$\rho(\text{kg/m}^3)$ :	8166	$\rho(\text{kg/m}^3)$ :	2370
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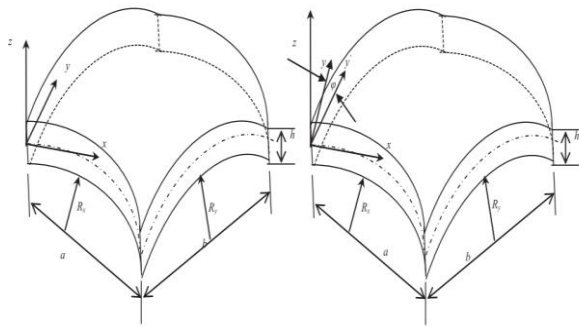


Fig. 7.(i) spherical curved shell form (ii) spherical curved shell skew form

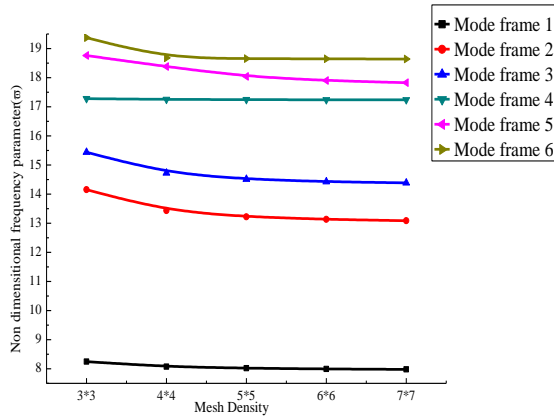


Fig. 8. Dimensionless frequency parameter SSSS

#### IV. RESULTS AND DISCUSSION

The theoretical model is developed in the present study using the HSDT and FG spherical shells under FV analysis. Material Properties play a vital role in determining the response of natural frequencies of the FG Spherical shell. Table I. shows the material properties of FGM constituents. It should be that the

non-dimensional excitation frequency parameter defined. The results presented in Table II-XII are shown based on the dimensionless natural frequency, according to, as in (16).

$$(\bar{\omega} = \omega(a^2/h)\sqrt{\rho_c/E_c}) \quad (16)$$

Table I. Material properties of Strain less steel and Silicon nitrate[10]

Metal:	SUS304	Ceramic:	Si <sub>3</sub> N <sub>4</sub>
E(Gpa):	207.78×10 <sup>9</sup>	E(Gpa):	322.27×10 <sup>9</sup>
$\mu$ :	0.3177	$\mu$ :	0.24

Table- II. shows the comparison of Non-dimensional frequency parameter for FG spherical (Aluminum (Al) and Silicon Carbide (SiC)), shell panels with different Thickness (a/h) ratios.

		Skew angle( $\alpha$ )									
		0 <sup>0</sup>		15 <sup>0</sup>		30 <sup>0</sup>		45 <sup>0</sup>		60 <sup>0</sup>	
a/h	Mode	Mouli et.al [26]	Present	Mouli et.al [26]	Present	Mouli et.al [26]	Present	Mouli et.al [26]	Present	Mouli et.al [26]	Present

#### 4.1 Convergence test

The convergence study of the present model analyzed to determine the uniform mesh size at which the natural frequencies converge and a suitable number of homogeneous layers to represent the FG Spherical panel. In the following, as a first part, to examine the present solution, the convergence properties of the fundamental frequency SUS304/ Si3N4 for spherical shells in the rectangular form with simply supported conditions considered. The results are In Fig.8. The discussion is made only on the present numerical results for the uniform mesh size of 6×6. Therefore, based on the above analysis, the subsequent investigations are carried out using a uniform mesh size of 6×6.

#### 4.2 Comparison Study

Table II. shows the frequencies of the first five modes for a Simply- supported, spherical shell (Rx = Ry = R) in rectangular form. Composed of Aluminum (Al) and Silicon Carbide (SiC) for four different values of the thickness ratios (a/h = 5,20,50, and 200).The power-law distribution parameters FGM (u=1, v=0, w=2,  $\phi$ =2), Geometrical parameters R/a=5, a/b=1 and five skew angle 0<sup>0</sup>, 15<sup>0</sup>, 30<sup>0</sup>, 45<sup>0</sup>, 60<sup>0</sup> are considered. The results of good agreement with the HSDT accurate solutions given by Mouli et al. [26] The maximum difference between them is 1.82% and by using the customized computer code in the MATLAB environment, which is developed based on HSDT formulated by the finite element approach. Some numerical examples solved to show the effectiveness of the present developed model.

#### 4.3 Numerical examples and results

In this section, the FV behavior of the FG spherical shell, which is composed of two constituents ceramic- rich (silicon nitride Si3N4), and metal-rich (stainless steel SUS304), and the material properties same for all numerical examples, and the Poisson's ratios ( $\mu$ ) of both materials 0.3 chosen. The results were analyzed using four-parameter power-law distribution. The material distribution applied through-thickness( $\xi$ ) direction. Power-law Variation and choice of power-law material distribution parameters (u, v, w). Examine the influence of the vibration behavior of the FG spherical shell with the effects of the material composition in terms of volume fraction. Four power-law distribution parameters FGM (u, v, w,  $\phi$ ) as in (3) and the significance of geometrical ratios, and skew angle on the non-dimensional frequency responses are FGM structure presented.

5	1	3.4100	3.4723	6.8281	6.7500	6.8290	6.7509	7.4322	7.5446	7.4328	7.5452
	2	3.6408	3.7063	7.0037	6.9254	7.0591	6.9807	7.3561	7.4637	8.2882	8.4083
	3	4.4156	4.4915	7.6278	7.5454	7.7858	7.7095	8.0807	8.1815	10.1929	10.3287
	4	6.1069	6.2049	8.9230	8.8249	9.3418	9.2572	9.9786	10.0878	11.8894	11.7596
	5	10.0314	10.1768	11.6240	11.4933	12.7402	12.6242	14.4919	14.5521	14.7162	14.6343
20	1	4.6859	4.7087	9.8392	9.8906	9.8399	9.8913	15.1368	15.2157	19.2254	19.3233
	2	5.2709	5.2962	10.1067	10.1585	11.6470	11.7064	15.8461	15.9268	20.6338	20.7377
	3	6.6948	6.7273	11.8048	11.8646	15.7501	15.8288	18.1545	18.2445	25.3188	25.4429
	4	9.5462	9.5934	15.6993	15.7782	23.3829	23.4959	23.9133	24.0316	32.7678	32.9211
	5	17.1977	17.2814	26.1756	26.3045	37.2351	37.4083	44.4248	44.6480	47.0353	46.9039
50	1	7.9556	7.9688	11.9952	12.0188	11.9964	12.0200	16.9843	17.0197	21.5536	21.5990
	2	8.8234	8.8384	12.8408	12.8659	15.2141	15.2436	19.1813	19.2201	23.5513	23.6006
	3	10.0229	10.0410	14.7504	14.7800	19.9778	20.0184	22.5151	22.5611	29.7915	29.8538
	4	12.7876	12.8121	19.3674	19.4070	29.6364	29.6978	29.9588	30.0198	44.0315	44.1225
	5	21.6786	21.7218	33.2335	33.3015	51.2778	51.3790	56.5794	56.6961	75.5599	75.7033
200	1	28.2112	28.2218	29.7181	29.7298	29.7252	29.7368	32.0759	32.0892	35.0713	35.0865
	2	28.7029	28.7137	30.3673	30.3793	32.3829	32.3963	34.6465	34.6614	36.9586	36.9751
	3	29.1911	29.2023	31.2642	31.2769	34.8612	34.8764	36.6680	36.6830	41.9366	41.9563
	4	30.6427	30.6547	34.2175	34.2323	42.4027	42.4227	43.4188	43.4394	55.7493	55.7774
	5	36.8841	36.9000	47.0289	47.0518	69.3334	69.3690	70.5579	70.5947	101.8152	101.8692

**4.3.1 Example**

Table III. shows the effect of general boundary conditions, namely- supported (SSSS), clamped boundary condition (CCCC), and Hinged (HHHH) all sides and FGM Power-law distribution( $u=1/v=1/w=2/\phi=2$ ) and the following geometric values used for the analysis ( $a/h=10, R/a=5$ ). The results reveal the SSSS condition exhibit lower frequencies for all six modes compared to other boundary conditions. Clamped boundary conditions would also strengthen for all frequency responses. Consequently, it induces higher frequency responses of vibration of the shell. A similar trend mirrored for all the other cases.

**Table-III: Variation of non-dimensional frequency responses With ( $\alpha$ ) for FG spherical shell (SUS304/Si3N4) panel for different boundary conditions.**

BCC	Mode	Skew angle ( $\alpha$ )				
		0°	15°	30°	45°	60°
SSSS	1	6.6053	7.1838	9.0387	13.0503	22.9678
	2	15.1406	15.0826	17.0006	21.9219	34.2981
	3	15.1419	17.2313	22.0854	27.0449	35.4864
	4	20.5923	21.1368	23.0593	28.4055	39.1695
	5	20.5951	21.3085	23.5884	31.0722	45.2798
	6	23.0281	23.3723	25.4397	32.1331	45.6577
CCCC	1	11.1883	11.7655	13.8032	18.6021	30.6740
	2	20.7881	20.6460	22.6770	28.2155	42.2235
	3	20.7881	22.9958	28.1145	37.8034	53.7746
	4	29.0141	29.4013	31.6162	38.7717	58.9064
	5	34.7061	36.2572	40.9409	46.1865	63.2556
	6	35.0660	37.6715	41.5800	48.5456	66.5826
HHHH	1	7.0277	7.5702	9.3421	13.2590	23.0909
	2	15.1740	15.1275	17.0526	21.9776	34.3717
	3	15.1752	17.2516	22.1042	31.1155	45.5663
	4	23.0613	23.4081	25.4796	32.1119	55.4464
	5	28.3663	29.9099	35.1487	41.5272	56.5141
	6	28.3730	31.0243	35.3998	44.7038	58.4264

**4.3.2 Example**

The effect of curvature ratio ( $R/a$ ), on the variation of the non-dimensional frequency parameter with four-parameter power-law distribution FGM ( $u=1/v=0/w=0/\phi=5$ ) and FG

spherical shell made of SUS304/Si3N4, is described with SSSS CCCC, and HHHH boundary conditions.

In Tables IV-VI, respectively the frequency response shown in these three tables. For the above cases, five different values of curvature ratio ( $R/a = 5, 10, 20$  and  $50, 100$ ) and Skew angle range from  $0^0$  to  $60^0$  are considered.

Tables IV and V show the frequencies for the first six modes for simply supported and clamped boundary conditions ( $a/h=10, a/b=1$ ), respectively. it is observed that the natural frequencies in all six modes increase with rising the skew angle. Whereas for shells with curvature ratio grows resultant shell stiffness change, the natural frequencies rates gradually dropped. Although the curvature ratio of any curved shell panel increases consistent approaches to the flatness.

Table VI. shows the non-dimensional frequency rate of the first six modes observed for accelerating trend HHHH boundary condition, with rising skew angle. Descent flow of non-dimensional frequency parameters while increasing curvature ratios.

**4.3.3 Example**

Tables VII to IX illustrate the effect of the aspect ratio on the frequency responses of the FG spherical shell panel has analyzed. for three different boundary conditions, and following geometric quantiles are five different values of aspect ratio ( $a/b = 1, 1.5, 2, 2.5, 3$ ) and the  $a/h=5, R/a=5$  and power-law distribution FGM ( $u=1/v=1/w=2/\phi=2$ ) with five skew angle  $0^0, 15^0, 30^0, 45^0, 60^0$ .

The detailed numerical results depicted in Table VII to IX. describe the variation of the fundamental natural frequencies with the power-law distribution, Skew angles for under different boundary conditions, the trend can be seen in numerical results that the non-dimensional frequencies rate slowly improves as the skew angle increases from  $0^0$  to  $30^0$ .



Pronounced frequency increments occurring when the skew angle rises from 30° to 60°. The result shows that the frequency parameter values are increasing with the aspect ratio. Because the large aspect ratios are comparatively stiffer, and it also prominent affect the structural design and vibration behavior of the structure.

Tables VIII and IX. illustrate the first six frequency modes for Variation of Non-dimensional frequency parameter for clamped condition and hinged condition. The frequencies gradually increase as the aspect ratio increases. Between them CCCC boundary condition exhibit higher frequency parameter values compared to the other boundary condition.

**4.3.4 Example**

Tables X to XII demonstrates the effect of the thickness ratio on the frequency responses of the FG spherical shell SUS304/Si3N4 panel. With FGM ( $u=1/v=0/w=2/ \phi=2$ ) the following geometric parameters ( $a/b=1, R/a=5$ ) and different thickness ratio values of ( $a/h=5,10,20,50,100,200$ ) are

presented. Increase the skew angle from 0° to 60°, the interval of 15° under three different boundary conditions considered.

Table X shows the frequencies in the first six modes for simply supported with the power-law distribution. The frequency parameters are showing the enhancing type of behavior with the increasing thickness ratio and skew angle. The trends of nondimensional frequency response attributed gradually intensify the first mode to the sixth mode. When the calculations performed with skew angle 0° to 60°, generally, the maximum frequency responses occurred at 60°. in which the without skew angle 0° represents a rectangular spherical shell. The response of the frequency parameter for the skew angle with the thickness ratio increases the rate of stability for all the FGM skew structures.

Table XI and XII shows similar approach the effect of thickness ratio increases with skew angle and power law distribution parameters kept constant non dimensional frequency parameter responses shown for clamped and hinged boundary condition. Increment trends noticed at skew angle more than 30°.

**Table-IV, V and VI: Variation of non-dimensional frequency responses with skew angle ( $\alpha$ ) for FG spherical shell (SUS304/Si3N4) panel for different curvature ratios(R/a).**

**Table-IV: (R/a) ratios under SSSS boundary condition**

R/a	Mode	0°	15°	30°	45°	60°
5	1	8.1342	8.8455	11.1265	16.0583	28.2408
	2	18.6354	18.5634	20.919	26.9614	42.1294
	3	18.6371	21.2048	27.1647	33.3529	43.7535
	4	25.4005	26.0718	28.4416	35.0249	48.2873
	5	25.4039	26.2838	29.0977	38.1963	55.6096
	6	28.329	28.751	31.2875	39.5096	56.2522
10	1	7.852	8.5889	10.9303	15.9359	28.1964
	2	18.54	18.4677	20.8429	26.9226	42.1646
	3	18.5416	21.1294	27.1325	33.3257	43.704
	4	25.3819	26.0521	28.4192	35.0052	48.2337
	5	25.3853	26.2626	29.0588	38.188	55.7096
	6	28.286	28.7097	31.2562	39.484	56.134
20	1	7.7809	8.5248	10.8825	15.9081	28.1906
	2	18.5194	18.447	20.8277	26.918	42.184
	3	18.521	21.1142	27.1297	33.3126	43.6821
	4	25.3726	26.0424	28.4084	34.9941	48.21
	5	25.376	26.2524	29.0431	38.1929	55.7586
	6	28.28	28.7043	31.2538	39.4836	56.0804
50	1	7.7618	8.5078	10.8705	15.9022	28.1924
	2	18.5157	18.4433	20.8259	26.9201	42.1961
	3	18.5173	21.1124	27.1323	33.305	43.67
	4	25.367	26.0366	28.4019	34.9871	48.1968
	5	25.3704	26.2464	29.0349	38.1988	55.7858
	6	28.2815	28.7059	31.2567	39.4873	56.0519
100	1	7.7594	8.5058	10.8692	15.9021	28.1939
	2	18.5159	18.4436	20.8265	26.9215	42.2002
	3	18.5175	21.113	27.1339	33.3025	43.6661
	4	25.3651	26.0347	28.3998	34.9847	48.1925
	5	25.3686	26.2445	29.0324	38.2013	55.7941
	6	28.2828	28.7073	31.2584	39.4892	56.0433

**Table-V: (R/a) ratios under CCCC boundary condition**

R/a	Mode	0°	15°	30°	45°	60°
5	1	13.7852	14.4958	17.0038	22.9074	37.7452
	2	25.5709	25.3974	27.8927	34.6932	51.8779
	3	25.5709	28.2801	34.5590	46.4707	66.0626
	4	35.6781	36.1550	38.8760	47.6140	72.5375
	5	42.6532	44.5548	50.3726	56.8799	77.5000
	6	43.1177	46.3126	51.0773	59.7193	81.8035
10	1	13.4253	14.1547	16.7163	22.6985	37.6241
	2	25.5091	25.3276	27.8275	34.6463	51.8646
	3	25.5091	28.2381	34.5472	46.4393	66.0542
	4	35.6406	36.1137	38.8344	47.6463	72.4524
	5	42.6497	44.5559	50.4235	56.8535	77.6167
	6	43.0799	46.2858	51.0933	59.6466	81.7925
20	1	13.3344	14.0689	16.6446	22.6473	37.5958
	2	25.4983	25.3145	27.8159	34.6402	51.8695
	3	25.4983	28.2329	34.5514	46.4377	66.0605
	4	35.6369	36.1089	38.8297	47.6655	72.4136
	5	42.6568	44.5645	50.4383	56.8378	77.6706
	6	43.0762	46.2856	51.1071	59.6299	81.7971
50	1	13.3094	14.0453	16.6251	22.6338	37.5892
	2	25.4983	25.3137	27.8157	34.6421	51.8761
	3	25.4983	28.2350	34.5572	46.4414	66.0676
	4	35.6396	36.1112	38.8320	47.6780	72.3915
	5	42.6638	44.5723	50.4424	56.8276	77.7015
	6	43.0789	46.2896	51.1173	59.6262	81.8030
100	1	13.3060	14.0422	16.6226	22.6322	37.5887
	2	25.4994	25.3146	27.8167	34.6437	51.8789
	3	25.4994	28.2365	34.5596	46.4434	66.0705
	4	35.6413	36.1128	38.8336	47.6824	72.3844
	5	42.6666	44.5753	50.4429	56.8240	77.7115
	6	43.0806	46.2916	51.1210	59.6260	81.8056

**Table-VI: (R/a) ratios under HHHH boundary condition**

R/a	Mode	0°	15°	30°	45°	60°
5	1	8.4725	9.1523	11.3585	16.2029	28.3116
	2	18.6498	18.5873	20.9517	27.0024	42.2158

**Table-VII: (a/b) ratios under SSSS boundary condition**

a/b	Mode	0°	15°	30°	45°	60°
5	1	5.7832	6.1551	7.4093	10.1490	16.4495
	2	10.2960	10.5675	11.5274	13.5194	17.7337









Based on the parametric study, numerical results revealed by using the four-parameter Power-law distribution on rectangular spherical shell and influence of  $u$ ,  $v$ , and  $w$  parameters on the non-dimensional frequency responses examined. The following important conclusion from this study is summarized.

- The frequency responses are decreasing trend as the curvature ratio increases.
- The frequency responses are gradually rising as the thickness ratio and aspect ratio increases.
- In all cases, the boundary conditions have a perceptible effect on the frequency parameter of the spherical shell. The frequency parameters are higher for all the FG shell clamped boundary conditions.
- Variation of frequency parameter with a different power-law exponent ( $\phi$ ) and material parameters ( $u$ ,  $v$ , and  $w$ ) It is possible to approach to change the behavior of the structure.
- The influence of skewangles the non-dimensional frequency parameters exhibit 00 for minimum at 600 maximum response. all the frequency parameter values are escalating with skewangles.
- The frequency responses are maximum and minimum for the spherical shells.

## V. SUMMARY AND CONCLUSIONS

In this study, free vibration responses of FGM spherical shell (SUS304/(Si3N4) rectangular form observed and presented. The FGM material composition, material properties vary continuously from metal (top/bottom surface) to ceramic (base/top surface). Employed Voigt's micromechanical model achieved through the four-parameter power-law distribution of the volume fractions. Convergence and comparison tests performed to illustrate the stability and exactness of the present mathematical model governed by

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